ST. HENRY'S COLLEGE KITOVU

A'LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2018

STATISTICS AND PROBABILITY

1. a) Events A and B are such that $P(A)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{5}$ and $P(A^1\cap B^1)=\frac{1}{6}$. Find;

   i) $P(A\cup B)$,

   ii) $P(A/B^1)$.

   b) There are 3 black and 2 white balls in each of 2 bags. A ball is taken from the first bag and put in the second bag. A ball is then taken from the second and put in the first bag. What is the probability that there is now the same number of black and white balls in each bag as there were to begin with?

2. a) The chance that Moses wins a game is $\frac{1}{3}$. If he plays nine games in a row, what is the;

   (i) expected number of games,

   (ii) chance of winning at least two games.

   b) At a bottle manufacturing factory, the new machine approximately makes 19% of the bottles that are damaged. If a random sample of 400 bottles is taken, find the probability that;

   (i) more than 31 bottles will be damaged,

   (ii) between 30 and 40 bottles inclusive will be damaged.

3. The table below show the frequency distribution of marks obtained by a group of students in a paper two mathematics examination.

Marks(%)	10 –	20 –	35 –	45 –	65 –	80 –	90 –
Frequency density	1.8	2.4	5.8	3.3	1.2	0.4	0

   a) Calculate the;

   i) modal mark,

   ii) mean mark,

   iii) standard deviation,

   iv) number of students who scored above 54%.

   b) Draw a cumulative frequency curve and use it to estimate the;

   i) P_{10}-P_{60} range,

   ii) number of students who scored below 40%,

   iii) least mark if 20% of the students scored a distinction.

4. The table below shows the cost of ingredients used for making Chapattis for two different birthday parties for 2016 and 2017.

Ingredients	**Cost**	
	2016	**2017**
Salt	200	350
Baking flour	3800	4600
Cooking oil	1500	1800

By taking 2016 as a base year, calculate the price relative for each ingredient and hence, obtain the average index number.

5. The table below shows marks scored by 8 students in mock and UNEB final examinations in Applied Mathematics.

Mock Examination	79	67	52	71	97	55	41	86
UNEB Final Examination	75	60	45	55	85	43	30	70

a) (i) Draw a scatter diagram for the data and comment on your result.
(ii) On the same diagram draw a line of best fit.
(iii) Use the line of best fit to estimate the mark that a student who scored 68 in Mock will score in UNEB.

b) Calculate the rank correlation coefficient for the marks in Mock and UNEB and comment on your result.

6. A random variable X takes the integer value x with P(x) defined by

$$P(X=1)=P(X=2)=P(X=3)=kx^2, P(X=4)=P(X=5)=P(X=6)=k(7-x)^2$$
.

Find the;

a) value of the constant k, hence sketch the graph of $f(x)$.

b) E(Y) and Var(Y) where $Y=4X-2$.

7. The continuous random variable X is such that $X \sim R(a,b)$. The lower quartile is 5 and the upper quartile is 9.

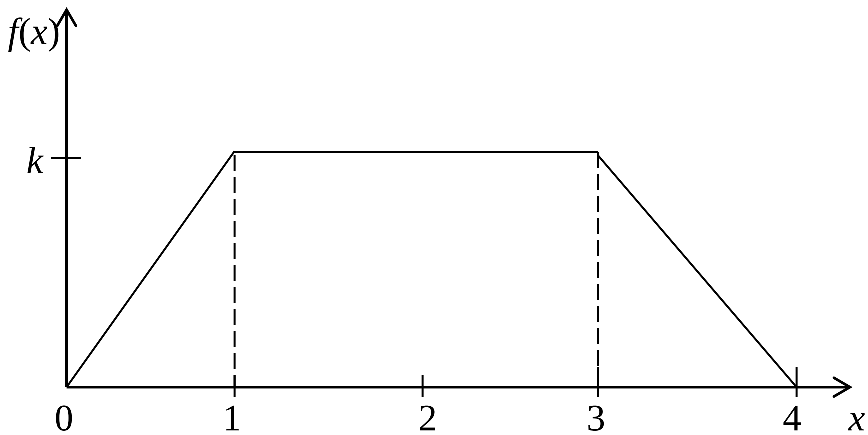
Find;

a) the values of a and b,

b) P(6<X<7),

c) the cumulative distribution function F(x).

8. The probability distribution function of a continuous random variable **X** is represented as shown.



Find the;

(i) mean of X,

(ii) value of *k*,

(iii) expression for the distribution,

(iv) $P(X \leq x)$, hence the median,

(v) P(1 .8 < x < 3.2).

9. The continuous random variable Y has a cumulative distribution function given by;

$$F(y) = \begin{cases} 0; & y < 1 \\ Ay^2(y^2 - 1); & 1 \leq y \leq 2 \\ 1; & y > 2 \end{cases}$$

Find the:

(a) value of A,

(b) 90^{th} Percentile,

(c) f(y), probability density function of Y,

(d) E(Y)

(e) Var(2Y+3)

10. The positive error made by a machine while manufacturing metal strips is a random variable which can take up any value up to 0.5 cm. it is known that the probability of the length being not more than y centimeters $(0 \leq y \leq 0.5)$ is equal to ky. Determine the;

(a) value of k,

(b) median positive error,

(c) probability distribution function,

(d) expected value of y,

(e) standard deviation correct to 3.s.f.

11. On day one of the Coachella music festival, the height of the revelers can be modeled into a normal distribution of mean 1.75m and variance 0,0064m^2. A draw is to be carried out and it is decided that one should have a height greater than 1.67m but less than 1.83m to participate.

(a) Find the:

(i) percentage of the people who qualify to take part in the draw.

(ii) fraction that is rejected because they are too tall.

By day three of the event, the heights of the people present are normally distributed with mean, μ, and standard deviation, 0.085m. When the criteria used to select participants is not altered, 3.5% of the revelers are rejected because they are too short.

(b) Find the:

(i) value of μ,

(ii) probability that a reveler whose height exceeds the mean qualifies to take part in the draw.

12. The germination time of a certain species of beans is known to be normally distributed. In a given bath of these beans, 20% take more than 6 days to germinate and 10% take less than 4 days.

(a) Determine the mean and standard deviation of the germination time.

(b) Find the 99.5% confidence limits of the germination time.

MECHANICS

13. (a) A particle has an initial position vector of $(4i + 3j - 7k)m$. The particle moves with a constant velocity of $6ms^{-1}$ parallel to $2i - j + 2k$. Find the position vector of the particle $3s$ later. Hence find how far the particle is from the origin.

(b) A particle A of mass 2 kg moves under the action of a constant force $(2i + 4j + 3k)N$. At $t = 0s$, the particles is stationary and at a point with position vector $(4i - 10j + 7k)m$. Find the position vector of the particle at time $t = 5s$.

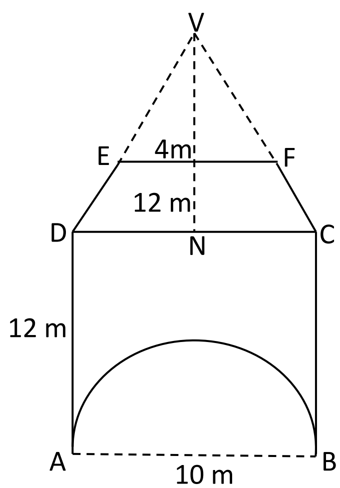
14.(a) An aircraft, at a height of 180m above horizontal ground and flying horizontally with a speed of $70ms^{-1}$, releases emergency supplies. If these supplies are to land at a specific point, at what horizontal distance from these point must the aircraft release them?

(b) A particle is projected from a point on a horizontal plane and has an initial speed of $42ms^{-1}$. If the particle passes through a point above the plane, $70m$ vertically and $60m$ horizontally from the point of projection, find the possible angles of projection.

15.(a) A ball is projected from a point O with an angle of projection α. Find α if the horizontal range of the particle is five times the greatest height reached by it.

(b) A stone thrown upwards from the top of a vertical cliff $56m$ high falls into the sea 4s later, $32m$ from the foot of the cliff. Find the speed and direction of projection. (Take g to be $10ms^{-2}$)

16.



ABCD is a uniform rectangular lamina with AB = $10m$ and AD = $12m$. A semi-circle of diameter AB is cut off. An isosceles uniform triangular lamina of base equal to CD, EF = 4m, NV = 12m and EFV cut off is joined at side CD as shown above.

(a) Calculate the centre of gravity of the remaining system.

(b) If the system is then suspended at A, find the angle side AD makes with the downward vertical.

17. A non-uniform ladder AB of length 10m, weighing $5W$ and centre of mass $4m$ from A rest in a vertical plane with end B against a rough vertical wall and the end A against a rough horizontal surface. If the coefficient of friction at each end is $\frac{1}{4}$ and $\frac{1}{2}$ respectively.

(a) A man of weight $13W$ begins to ascend the ladder from the foot, find how far he will climb before the ladder slips?

(b) If a horizontal inextensible string is the attached from end A to the base of the wall, find the tension in the string when the man climbed the ladder to end B.

18. Two airfields A and B are $150km$ apart with B on a bearing of 045^0 from A. A wind of $30kmh^{-1}$ is blowing from a direction 260^0. Assuming this wind remains constant throughout. Find the time required for aircraft to fly from A to B and back to A again, if the aircraft can fly at $100kmh^{-1}$ in still air.

19. A helicopter sets off from its base and flies at $50ms^{-1}$ to intercept a ship which, when the helicopter sets off, is at a distance of 5km on a bearing of 335^0 from the base. The ship is travelling at $10ms^{-1}$ on a bearing of 095^0. Find;
(a) The course that the helicopter pilot should set if he is to intercept the ship as quickly as possible.
(b) The time interval between the helicopter taking off and it reaching the ship.

20. At 11:45 am, a trawler is 10km due east of a launch. The trawler maintained a steady $10kmh^{-1}$ on a bearing 180^0 and the launch maintains a steady a steady $20kmh^{-1}$ on a bearing 071^0.
(a) Find the minimum distance the boats are apart in the subsequent motion, and the time at which this occurs.
(b) Find to the nearest minutes, the length of time for which the two boats are within 8km of each other.

21. ABCD is a rectangle with AB = $4m$ and AD = $3m$. Forces of $9N$, $8N$, $4N$, $7N$ and $10N$ act along AB, CB, CD, AD and BD respectively with the direction indicated by the order of the letters.
(a) Calculate the magnitude and direction of the single force that could replace this system of forces.
(b) Find the equation of the line of action of the single force and where the line cuts AB.

22.(a) Find the coordinates of centre of gravity of a uniform lamina which lies in the first quadrant and is enclosed by the curves $y = 3x^2,\ y = 4 - x^2$ and the y – axis.
(b) Two rods AB and BC are joined together at B such that $A\hat{B}C = 50^0$. AB is uniform, of length $7m$ and mass $5kg$. BC is uniform, of length 6m and mass $5kg$. Find the distance of the centre of gravity from B.

23.(a) A small object of weight $4w$ in rough contact with a horizontal plane is acted upon by a force inclined at 30^0 to the plane. When the force is of magnitude $2w$, the object is about to slip. Calculate the magnitude of the normal reaction and coefficient of friction between the object and the plane.

(b) A particle of mass $5kg$ is placed on a rough plane which is inclined at 30^0 to the horizontal. The angle of friction between the particle and the plane is 14^0. Find the horizontal force that should be applied to the particle so that;
   (i) The particle is just prevented from sliding down the plane.
   (ii) The particle moves up the plane with an acceleration of $3.2ms^{-1}$.

24. A body A of mass $2kg$ is moving with a velocity $(-2i + 3j)ms^{-1}$ when it collide with a body B of mass $5kg$, moving with a velocity $(6i - 10j)ms^{-1}$. Immediately after the collision the velocity of A is $(3i - 2j)ms^{-1}$. Find;
   (a) The velocity of B after the collision
   (b) The loss in kinetic energy of the system due to the collision
   (c) The impulse of A on B due to the collision.

25.(a) A particle is attached to one end of a light inextensible string which has its other end attached to a fixed point A. with the string taut, the particle describes a horizontal circle with a constant angular speed 2.8 rad s^{-1}, the centre of the circle being at a point O vertically below A. find the distance OA.

   (b) A vehicle is just on the point of slipping when parked on a bent that is banked at an angle of 20^0 to the horizontal.
   (i) Find the coefficient of friction between the vehicle tyres and the surface of the road
   (ii) If the vehicle were driven around this bend in a horizontal circular path of radius $60m$, find the greatest speed it could attain without slipping occurring.

26. A water pump raises $40kg$ of water a second through a height of $20m$ and ejects it with a speed of $45ms^{-1}$.
   (a) Find the kinetic energy and potential energy per second given to the water
   (b) Calculate the effective rate at which the pump is working

27. With its engine working at a constant rate of $18k$W, a vehicle of mass 1.5 tones ascents a hill of 1 in 98 against a constant resistance to motion of $450N$. find;
   (a) The acceleration of the vehicle up the hill when travelling with a speed of $10ms^{-1}$.
   (b) The maximum speed of the vehicle up the hill.

28.(a) A particle performs a SHM of periods 4s and amplitude $2cm$ about a centre O. find the time it takes the particle to travel from O to a point P, a distance $\sqrt{2}\ cm$ from O.

(b) A particle moves with SHM about a mean position O. when passing through two points which are $2m$ and $2.4m$ from O the particle has speeds of $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the amplitude of the motion and the greatest speed attained by the particle.

NUMERICAL METHODS

29.(a) Given that $x = 2.40$, $y = 5.613$ and $z = 8.446$, each number rounded off to the given number of decimal places. Find the;

(i) Limits within which the exact value of $\frac{x(4.5-z)}{y}$ lies

(ii) Percentage error made in calculating $\frac{z-y}{x}$. (give your answer correct to 2dps)

(b) Two decimal numbers X and Y were rounded off to give x and y with errors e_x and e_y respectively. Show that the maximum absolute and relative errors made in approximating $XY^{\frac{1}{2}}$ by $xy^{\frac{1}{2}}$ are given by $\left|y^{\frac{1}{2}}e_x\right| + \left|\frac{xe_y}{2y^{\frac{1}{2}}}\right|$ and $\left|\frac{e_x}{x}\right| + \frac{1}{2}\left|\frac{e_y}{y}\right|$ respectively.

30.(a) Use trapezium rule with six ordinates to estimate $\int_0^{\frac{\pi}{3}} tanx\, dx$, correct to **three** decimal places.

(b) Calculate the error made the estimate in (a) above and suggest how that error can be reduced.

31.(a) The table below shows the values of a continuous function f with respect to x.

x	1	2	3	4
$f(x)$	-1.632	-0.865	0.050	1.018

Using linear interpolation or extrapolation, find;

(i) $f(x)$ when $x = 2.7$

(ii) $f^I(1.2)$.

(b) Show that the root of the equation $f(x) = e^{-x} + x - 3$ lies between 2 and 3. Hence use linear interpolation to find the root correct to **two** decimal places

32.(a) Show that the root of the equation $lnx + 2x - 3 = 0$ lies between 1 and 2.

(b) Show that the iterative formula based on Newton Raphson's method for solving the equation is (a) above is given by

$$x_{n+1} = \frac{x_n(4-lnx_n)}{1+2x_n}, \quad n = 0, 1, 2, ...$$

Hence find the root of the equation correct to **two** decimal places.

33.(a) Show that the Newton Raphson's formula for finding the fourth root of a number N is

$$x_{n+1} = \frac{1}{4}\left(3x_n + \frac{N}{x_n^3}\right),\ n = 0, 1, 2, \ldots$$

(b) Construct a flow chart that;

- Read N and the first approximation x_0
- Compute and print the root x_{n+1} and the number of iteration, n.

Using the flow chart, show that $(67)^{\frac{1}{4}} \approx 2.86$ to two decimal places.

END

ST. HENRY'S COLLEGE KITOVU

A'LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2019

STATISTICS AND PROBABILITY

1. (a) The data below was obtained from a survey carried out on the temperature variations of 10 cities in a cold season of the year.

$$0^oC, -2^oC, 3^oC, 4^oC, -4^oC, -10^oC, 5^oC, 1^oC, -8^oC, -7^oC.$$

Determine the;

(i) mean temperature,

(ii) variance.

(b) A sample of **n** members of the rotary club of Masaka was asked how many crates of beer they took in a given month.
The results were as follows $\sum \boldsymbol{x} = \mathbf{225}, \sum \boldsymbol{x}^{\mathbf{2}} = \mathbf{1755}$. Find the possible values of **n** if the standard deviation is **1.5** and hence find the respective mean number of crates taken by the Rotarians.

2. The table shows the marks scored by a group of candidates in a mathematics exam.

Marks (%)	10 - 19	20 - 29	30 - 34	35- 44	45-54	55-64	65-69
Frequency density	0.7	2.6	4.2	3.8	4.6	2.8	2.6

(a) Draw a histogram and use it to estimate the modal mark.

(b) Calculate the;

(i) mean mark,

(ii) median mark,

(iii) standard deviation.

3. The table below shows the marks obtained by a group of students in a math test.

Marks(%)	$20-29$	$30-34$	$35-44$	$45-64$	$65-74$	$75-84$
Frequency	5	5	12	20	10	8

Calculate the;

(a) mean mark,

(b) standard deviation,

(c) mode,

(d) median,

(e) semi-interquartile range,

(f) middle 60% of the marks.
(g) number of students whose marks exceed 47%.

4. The table below shows the heights (y) in centimetres and the age (x), in years, of a group of students in a certain secondary school.

Student	A	B	C	D	E	F	G	H	I	J	K	L
Age (x)	12	14	13	15	17	20	17	15	18	19	14	16
Height (y)	130	136	120	120	153	160	155	142	145	172	140	157

(a) Construct a scatter diagram, draw the line of best fit and comment hence estimate x when $y = 142$.
(b) Giving rank 1 to the tallest student and oldest student, calculate the rank correlation coefficient and comment at 5% level of significance.

5. A class of 10 students were examined in Economics (E) and Mathematics (M). The following table shows their scores out 10.

Student	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
E	7.2	5.2	3.1	3.8	8.1	5.2	4.0	6.0	6.3	7.5
M	6.4	6.0	6.5	4.3	7.0	4.8	3.4	6.2	5.9	6.0

(a) Plot the above on a scatter diagram.
(b) Draw a line of best fit and estimate the value of E when $M = 8.6$.
(c) Compute the rank correlation coefficient between E and M and comment on you result.

6. (a) The table below shows the expenditure (in Ug. shs) of a student during the first and second terms.

ITEM	EXPENDITURE		WEIGHT
	First term	Second term	
Clothing	46,500	49,350	5
Pocket money	55,200	57,500	3
Books	80,000	97,500	8

Using first term expenditure as the base, calculate the average weighted price index to **one** decimal place.

(b) The price relative of commodity in 2010, using 2009 as base year was 105. The price relative of same commodity in 2012 using 2010 as base year was 95. Given that the cost of the commodity in 2009 was shs. 259,250, find its cost in 2012.

(c) 2015 being the base year, the price index of a particular commodity in 2017 was 110 and if 2017 is used as the base year, the price index in 2018 is 120. Calculate the index number for 2018 taking 2015 as the base year.

7. (a) Two events A and B are such that $P(A) = 0.7$, $P(A \cap B) = 0.45$ and $P(A^1 \cap B^1) = 0.18$. Find;

(i) $P(B')$ (ii) $P(AorB)$

(b) M and N are two events such that $P(M) = 0.3$, $P(N) = 0.1$ and $P(M/N) = 0.2$, find:

(i) $P(M' \cup N')$, (ii) $P(M/N')$

(c) Events A, B and C are mutually exclusive and exhaustive such that $P(A) = k(1-2x), P(B) = k(1+x)$ and $P(C) = k(3+x)$. Find the value of

(i) k (ii) $P(B)$ if $x = \frac{1}{4}$.

(d) If two events C and D independent such that their chance of occurring together is $\frac{1}{5}$ and the chance that either C or D occurs is $\frac{7}{8}$.

(i) Show that C' and D' are also independent.

(ii) Find $P(\mathrm{C})$ and $P(\mathrm{D})$.

8. (a) A bag A contains 5 red ball and 3 black balls and bag B contains 3 red balls and a black ball. A bag is selected at random and two balls drawn from it without replacement. If A is twice as likely to be chosen as B,

(i) Find the probability that both balls are of different colours and the probability that the balls are from A given that they are of different colours.

(ii) Construct a probability distribution table for the number of black balls drawn. Hence find the mean number of black ball drawn and the variance.

(b) A box P contains 1 red, 3 green and 1 blue bead. Box Q contains 2 red, 1 green and 2 blue beads. A balanced die is thrown and if the throw shows a six, box P is chosen otherwise box Q is chosen. A bead is drawn at random from the chosen box. Given that a green bead is drawn, find the probability that it came from box P.

9. A random variable X has a p.d.f given by;

$$f(x) = kx; \quad 0 \leq x \leq 3, \quad f(x) = 3k(4-x); \quad 3 \leq x \leq 4$$

$$f(x) = 0; \ x < 0 \ and \ x > 4.$$

(a) Sketch $f(x)$ hence the value of k.

(b) Find $E(3X-1)$.

(c) Obtain the cumulative distribution, $P(X \leq x)$. Hence find

(i) $P(X < 3.5)$, (ii) the median and inter quartile range.

10.(a) The continuous random variable Y is uniformly distributed in the interval $a < Y < b$.

The lower quartile is 5 and the upper quartile is 9. Find

(i) the values of a and b, (ii) $P(6 < X < 7)$

(iii) the cumulative distribution function F(x).

(b) Given that $X \sim R[32, 37]$, find;

(i) E(X)

(ii) the probability that X lies within one standard deviation of the mean.

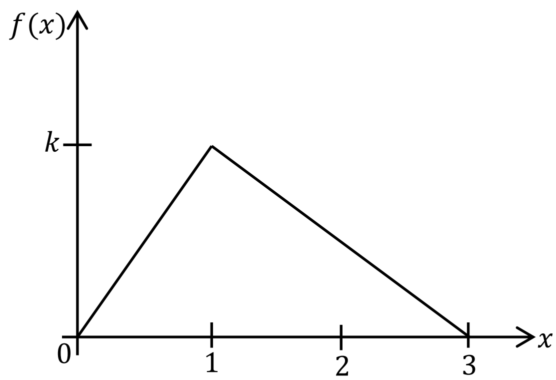
(c) A continuous random variable X has cumulative distribution function;

$F(\mathrm{X}) = \frac{x-2}{5}, \ 2 \leq \mathrm{X} \leq 7$

Find

(i) $P(\mathrm{X} < 6/\mathrm{X} > 3)$ (ii) $E(\mathrm{X})$ and $Var(\mathrm{X})$,

11. A random variable X has its probability function as shown below.



(i) Find the value of k.
(ii) Find the probability density function, $f(x)$.
(iii) Compute $P(|X-1| < \frac{1}{2})$
(iv) Derive the distribution function, F(x) and hence find the median of X.

12. (a) A couple is equally likely to produce a boy or a girl. Find the probability that in a family of five children there more boys than girls.

(b) A box contains equal number of red counters as yellow counters. A counter is taken from the box, its colour is noted the replaced. This is performed eight times in all. Calculate the probability that;

(i) exactly three will be red,

(ii) at least one will be red,

(iii) more than four will be yellow.

(c) A random variable X is such that $X \sim B(10, p)$ where $p < 0.5$ and $Var(X) = 1.875$. Find

(i) the value of p,

(ii) E(X),

(iii) $P(X = 2)$.

13. The masses of packets of sugar from a certain factory are normally distributed. In a large consignment of packets of sugar, it is found that 5% of them have a mass greater than 510 g and 2% have a mass greater than 515 g.

Estimate;

(a) the mean and standard deviation of this distribution,

(b) the probability that a packet picked at random from this consignment weighs more than 500 g.

14. The records from a Health Centre IV in Wakiso district showed that 80% of the patients who visited the centre on a certain day had malaria. Find the probability that on a particular day when 200 patients visited the centre;

(a) more than 70 patients tested positive for malaria,

(b) at least 55 patients were not suffering from malaria,

(c) less than 170 patients tested positive for malaria.

15. The heights of boys in a certain village follow a normal distribution with mean 150 cm and variance 25 cm^2. Find the probability that a boy picked at random from the village has height:

(a) less the 153 cm,
(b) more than 158 cm,
(c) between 149 cm and 159 cm,
(d) more than 10 cm difference from the mean height.

16. The masses of cows on Mr. Kato's farm of local cattle are normally distributed. It is discovered that 5% of the cows have a mass greater than 110 kg and 2% have a greater than 115 kg. Estimate the:
    (a) mean and standard deviation of this distribution,
    (b) number of cows in a group of 1000 cows which weigh between 109 kg and 121 kg.
    A veterinary doctor visited the farm and found that 10% of the cows malnourished. Determine the weight of the heaviest malnourished cow.

17.(a) The chance that a hen on Mrs. Musoke's poultry farm is infected with a deadly virus is 0.4. If a sample of 150 hens were inspected on the farm, find the 99.5% confidence limits for the mean number of cows that are infected.
   (b) A 95% confidence interval for the mean life of a particular type of smartphone battery was calculated and the confidence limits were 1023.3 hours and 1101.7 hours. The interval was based on a sample of 36 smartphone batteries. Find the 99% confidence interval for the mean life of this type of batteries.
   (c) A random variable of 50 readings taken from a normal population gave the following data: $\sum x = 163$ and $\sum x^2 = 548$. Calculate the:
      (i) unbiased estimate for the population variance,
      (ii) 98.46% confidence interval for the population mean.

MECHANICS

18.(a) ABC is an isosceles triangle, right angled at A with $\overline{AB} = 2$m. forces of 8N, 4N, and 6N act along the sides BA, CB, and CA respectively. Find the magnitude and direction of the resultant force.
   (b) In a square ABCD, three forces of magnitude 4N, 10N, and 7N act along AB, AD and CA respectively. Their directions are in the order of the letters. Find the magnitude of the resultant force.
   (c) In an equilateral triangle PQR, three forces of magnitude 5N, 10N, and 8N act along sides PQ, QR, and PR respectively. Their directions are in order of the letters. Find the magnitude of the resultant force.

19. A particle of mass 15kg is pulled up a smooth slope by a light inextensible string parallel to the slope. The slope is 10.5 m long and inclined at $\sin^{-1}(4/7)$ to the horizontal. The acceleration of the particle is $0.98ms^{-2}$.
Determine the:
(a) Tension in the string.
(b) Work done against gravity when the particle reaches the end of the slope

21. (a) A particle performs a SHM of periods 4s and amplitude $2cm$ about a centre O. find the time it takes the particle to travel from O to a point P, a distance $\sqrt{2}\ cm$ from O.
(b) A particle is moving with linear simple harmonic motion of amplitude 1.5m. The speed of the particle is $\sqrt{50}$ ms^{-1} when its displacement from the end point is 1m. Calculate its maximum acceleration.
(c) A particle moves with SHM about a mean position O. when passing through two points which are $2m$ and $2.4m$ from O the particle has speeds of $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the amplitude of the motion and the greatest speed attained by the particle.

22.(a) A body moving initially with a velocity u covers a distance x after t seconds. If it moves with a uniform acceleration a, derive an expression relating x, t, u, and a.
(b) A train approaching a station does two successive half kilometers in 16s and 20s respectively. Assuming a uniform retardation, calculate the further distance the train runs before it comes to rest.
(c) A body falls from rest from the top of a tower and during the last second it falls $\frac{9}{25}$ of the whole distance. Find the height of the tower.

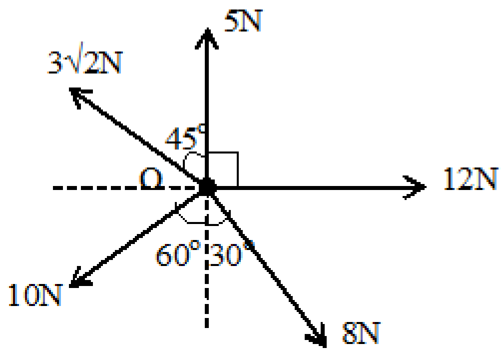
23. A body moving with acceleration $e^{2t}\boldsymbol{i} - 3\sin 2t\boldsymbol{j} + 4\cos 2t\,\boldsymbol{k}$ is initially located at the point (1, -2, 2)m and has a velocity of $4\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ ms^{-1}. Find the;
a) Speed of the body when $t = \frac{\pi}{4}$s.
b) Distance of the body from the origin at $t = \frac{\pi}{4}$

24. (a) Forces of 7N and 4N act away from a common point and make an angle of θ^o with each other. Given that the magnitude of their resultant is 10.75N, find the;

(i) Value of θ^o

(ii) Direction of the resultant force

(b) In the diagram below find the magnitude and direction of the resultant force



25. A boy throws a stone at a vertical wall a distance, d away. Given that R is the maximum range on the horizontal through the point of projection that can be attained by the speed of projection, show that;

(a) the height above the point of projection of highest point on the wall he can hit is $\left(\frac{R^2-d^2}{2R}\right)$,

(b) in this case, the angle of projection is $tan^{-1}\left(\frac{R}{d}\right)$.

26. A ball is hit at a point O, which is at a height of 2m above the ground and at a horizontal distance 4m from the wall, the initial speed being in a direction of 45^0 above the horizontal. If the ball just clears the wall which is 1m high,

(a) show that the equation of path of the ball is $16y = 16x - 5x^2$.

(b) calculate the;

(i) distance from the net at which the ball strikes the ground.

(ii) magnitude and direction of the velocity with which the ball strikes the ground.

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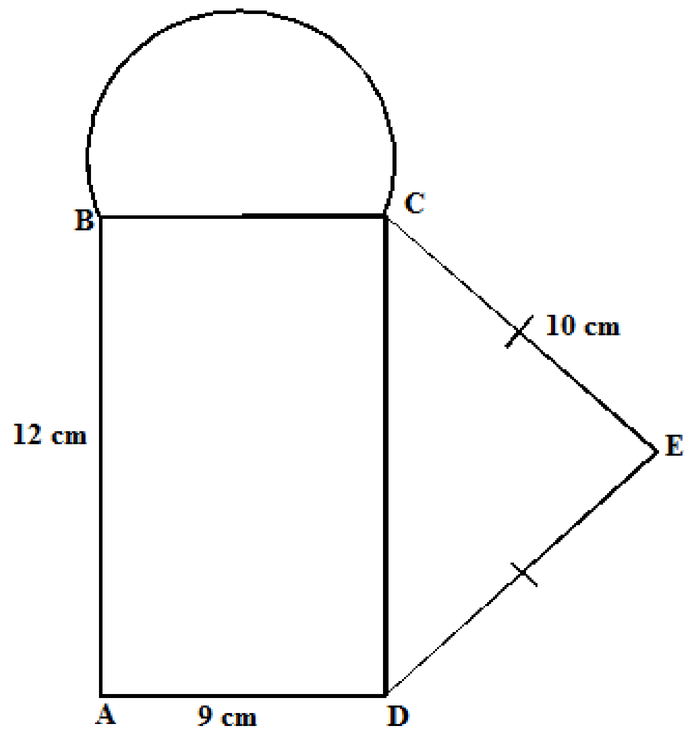
27. With its engine working at a constant rate of $18kW$, a vehicle of mass 1.5 tones ascents a hill of 1 in 98 against a constant resistance to motion of $450N$. find;

(a) The acceleration of the vehicle up the hill when travelling with a speed of $10ms^{-1}$.

(b) The maximum speed of the vehicle up the hill.

28. The figure below represents a lamina formed by welding together rectangular, semi-circular and triangular metal sheets.



Find the position of the centre of gravity of the lamina from the sides AB and AD.

29. At 10:00 am, ship A and ship B are 16 km apart. Ship A is on a bearing N35^{o}E from ship B. ship A is travelling at 14 kmh^{-1} on bearing S29^{o}E. ship B is travelling at 17 kmh^{-1} on a bearing N50^{o}E. determine the;

(a) Velocity of ship B relative to ship A

(b) Closest distance between the two ships and the time when it occurs

30. (a) Two ships A and B are observed from a coast guard station and have the following displacement velocities and times.

Ship	Displacement	Velocity	Time (t)
A	$(\boldsymbol{i} + 3\boldsymbol{j})$ km	$(\boldsymbol{i} + 2\boldsymbol{j})kmhr^{-1}$	12:00 hours
B	$(\boldsymbol{i} + 2\boldsymbol{j})$ km	$(5\boldsymbol{i} + 6\boldsymbol{j})kmhr^{-1}$	13:00 hours

Find the time when the two are closest to each other.

(b) If at 13:00 hours ship A changed it's velocity to $\left(\frac{11}{3}\boldsymbol{i} + 2\boldsymbol{j}\right) kmhr^{-1}$, show that they collide and find the time and position of collision.

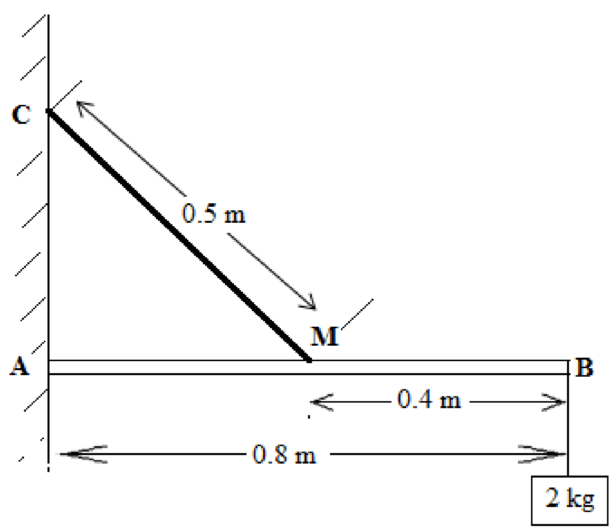
31. A light inextensible string has one end attached to a ceiling, the string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley, the particle of mass 5 kg is attached at the free end of the string, the sections of the strings not in contact with the pulleys are vertical, if the system is released from rest and moves in a vertical plane, determine the;

(a) Accelerations of the 2kg and 5kg masses

(b) Tensions of the 2kg and 5kg masses

(c) Distance moved by the system in 1.5 seconds.

32. The figure below shows a uniform beam of length 0.8 metres and mass 1 kg. The beam is hinged at A and has a load of mass 2 kg attached at B.



The beam is held in a horizontal position by a light inextensible string of length 0.5 metres. The string joins the mid-point M of the beam to a point C vertically above A. Find the:

(a) Tension in the string.

(b) Magnitude and direction of the force exerted by the hinge.

33. A non-uniform ladder AB of length 10m, weighing $5W$ and centre of mass $4m$ from A rest in a vertical plane with end B against a rough vertical wall and the end A against a rough horizontal surface. The angle between the ladder and the horizontal is 50^{o} and the coefficient of friction at each end is $\frac{1}{4}$ and $\frac{1}{2}$ respectively.

(a) A man of weight $13W$ begins to ascend the ladder from the foot, find how far he will climb before the ladder slips?

(b) If a horizontal inextensible string is the attached from end A to the base of the wall, find the tension in the string when the man climbed the ladder to end B.

34. A particle of mass 2 kg is acted upon by a force of 21N in the direction $2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$. Find in vector form the;

(a) Force

(b) Acceleration hence its magnitude.

35. Five forces of magnitudes 3N, 4N, 4N, 3N and 5N act along the lines AB, BC, CD, DA, and AC respectively, of a square ABCD of side 1m. The direction of the forces is given by the order of the letters. Taking AB and AD as reference axes; find the

(a) Magnitude and direction of the resultant force.

(b) Equation of the line of action of the resultant force and hence find the point where the resultant force cuts the side AB.

36. A particle of mass 4 kg starts from rest at a point $(2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$ m. it moves with acceleration $\boldsymbol{a} = (4\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})ms^{-2}$ when a constant force $\boldsymbol{F}$ acts on it. Find the:

(a) Force $\boldsymbol{F}$.

(b) velocity at any time t.

(c) work done by the force **F** after 6 seconds

35. At **10.00am**, ship **A** moving at **20km/hr** due east is **10km** South East of another ship ***B***. If ***B*** is moving at **14km/hr** in direction **S30^{0}W** and the ships maintain their velocities, find the;

(a) Time when the ships are **closest** together and the **shortest distance** between the ships.

(b) Bearing of **A** from ***B*** at that time.

36. A particle A initially at the point with position vector $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}\ km$ is moving with a constant velocity of $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}\ kmh^{-1}$. At the same instant, a particle B at the point $(3, 3, 2)$ is moving with a constant velocity of $3\mathbf{i} - 2\mathbf{k}\ kmh^{-1}$. Find the:

(i) relative velocity of particle A to B.

(ii) relative displacement of particle A to particle B at any instant.

(iii) shortest distance between the two particles in their subsequent motion.

37. (a) Two ships A and B are observed from a coast guard station and have the following displacement velocities and times.

Ship	Displacement	Velocity	Time (t)
A	$(\boldsymbol{i} + 3\boldsymbol{j})$ km	$(\boldsymbol{i} + 2\boldsymbol{j})kmhr^{-1}$	12:00 hours
B	$(\boldsymbol{i} + 2\boldsymbol{j})$ km	$(5\boldsymbol{i} + 6\boldsymbol{j})kmhr^{-1}$	13:00 hours

Find the time when the two are closest to each other.

(b) If at 13:00 hours ship A changed it's velocity to $\left(\frac{11}{3}\boldsymbol{i} + 2\boldsymbol{j}\right)kmhr^{-1}$, show that they collide and find the time and position of collision.

NUMERICAL METHODS

38. (a) Show that the iterative formula based on Newton Raphson's method for solving the equation $e^{2x} + 4x = 5$ is given by,

$$x_{n+1} = \frac{e^{2x_n}(2x_n - 1) + 5}{2e^{2x_n} + 4}, \text{ n = 0, 2, 3...}$$

(b) (i) Construct a flow chart that;

- reads the initial approximation x_0,
- computes, using the iterative formula in (a) and prints the root of the equation $e^{2x} + 4x - 5 = 0$, and the number of iterations when the error is less than 1.0 x 10^{-4}.

(ii) Perform a dry run of the flow chart when $x_0 = 0.5$.

39. The iterative formulae below are used for calculating the positive root of the $f(x) = 0$.

$$A: \ x_{n+1} = \frac{1}{3}\left(\frac{2x_n{}^3+12}{x_n{}^2}\right)$$

$$B: x_{n+1} = \sqrt{\left(\frac{x_n{}^3+12}{2x_n}\right)}$$

(a) Taking $x = 2$, use each formula twice and hence deduce the most suitable for solving $f(x) = 0$.

(b) Find the root of the equation $f(x) = 0$, correct to three decimal places.

(c) Find the equation whose root is in b) above.

40. (a) Show that the equation $\ln x = \sin x + 2$ has a root between $x = 3$ and $x = 4$. Use linear interpolation to estimate the initial approximation x_o to 1 decimal place.

(b) Using the x_o above and the Newton Raphson method find the root correct it 3 decimal places.

41. (a) The table below shows the values of x and their corresponding natural logarithm

x	5.0	5.2	5.4	5.7	6.0
Inx	1.609	1.647	1.686	1.740	1.792

Use linear interpolation or extrapolation to find

(i) $In\,(5.56)$,

(ii) $e^{1.575}$.

(b) A car consumed fuel amounting to shs 14,800, shs 15,600, shs 16,400 and shs 17,200 in covering distances of 10km, 20km, 30km and 40km respectively. Estimate the;

(i) cost of fuel consumed for a distance of 45km,

(ii) distance travelled if fuel of shs 16,000 is used.

42. (a) Two positive decimal numbers X and Y were approximated with errors E_1 and E_2 respectively. Show that the maximum possible relative error in the approximation of the product X^3Y^2 is $3\left|\frac{E_1}{X}\right| + 2\left|\frac{E_2}{Y}\right|$.

(b) Given that X = 5.64 and Y = 10.0, rounded off to the given number of decimal places, find the;

(i) maximum possible errors in X and Y,

(ii) percentage error made in the approximation of X^3Y^2.

43.(a) Given that $y = xsinx$ and $x = 2$, find the absolute error in y giving your answer correct to 3 significant figures.

(b) The numbers $x = 1.5, y = -2.85$ and $z = 10.345$ were all rounded off to the given number of decimal places. Find the range within which the exact value of $\frac{1}{x} - \frac{1}{y} + \frac{y}{xz}$ lies.

44. (a) Using trapezium rule with five strips evaluate $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}} dx$, correct to three decimal places.

(b) Find the exact value of $\int_3^4 \frac{1}{\sqrt{(x-1)^2-3}} dx$.

(c) Find the percentage error in the approximation in a) above and suggest how this error can be reduced.

END

ALGEBRA

1. Solve the simultaneous equations:
   (a) $x^2 + y^2 = 5$, $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{5}{4}$
   (b) $\dfrac{x}{y} + \dfrac{y}{x} = \dfrac{17}{4}$, $x^2 - 4xy + y^2 = 1$

2. Find the range of values of x for which
   (a) $\dfrac{2x+1}{x+2} > \dfrac{1}{2}$.
   (b) $|2x+1| > 7$

3. Resolve into partial fractions
   (a) $\dfrac{x^3 + x^2 + 4x}{x^2 + x - 2}$
   (b) $\dfrac{3x^2 + 8x + 13}{(x-1)(x^2 + 2x + 5)}$
   (c) $\dfrac{2x^3 + 2x^2 + 2}{(x+1)^2(x^2+1)}$

4. Solve the following equations:
   (a) $2^{3x+1} = 5^{x+1}$
   (b) $9^x - 4(3^x) + 3 = 0$
   (c) $\log_x 9 + \log_{x^2} 3 = 2.5$
   (d) $\sqrt{2x-1} - \sqrt{x-1} = 1$
   (e) $\begin{aligned} 2x + 3y + 4z &= 8 \\ 3x - 2y - 3z &= -2 \\ 5x + 4y + 2z &= 3 \end{aligned}$

5. Find:
   (a) The three numbers in arithmetic progression such that their sum is 27 and their product is 504
   (b) The three numbers in a geometrical progression such that their sum 39 and their product is 729.
   (c) The sum of the last three terms of a geometrical progression having n terms is 1024 times the sum of the first three terms of the progression. If the third term is 5, find the last term.

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(d) Prove by induction that $1^3+2^3+....+n^3=\frac{1}{4}n^2(n+1)^2$ and deduce that

$1^3+3^3+5^3....+(2n+1)^3=(n+1)^2(2n^2+4n+1)$

6. Expand:

(a) $\frac{7+x}{(1+x)(1+x^2)}$ in ascending powers of x as far as the term in x^4.

(b) $\left(1-\frac{3}{2}x-x^2\right)^5$ in ascending powers of x as far as the term in x^4.

(c) Find the term independent of x in the expansion of $\left(2x+1/x^2\right)^{12}$ in descending powers of x and find the greatest term in the expansion when $x=\frac{2}{3}$.

(d) Find by binomial theorem, the coefficient of x^8 in the expansion $(3-5x^2)^{1/2}$ in ascending powers of x.

(e) In the binomial expansion of $(1+x)^{n+1}$, n being an integer greater than two, the coefficient of x^4 is six times the coefficient of x^2 in the expansion $(1+x)^{n-1}$. Determine the value of n.

7. (a) Without using the calculator, simplify $\dfrac{\left(\cos\left(\frac{\pi}{9}\right)+i\sin\left(\frac{\pi}{9}\right)\right)^4}{\left(\cos\left(\frac{\pi}{9}\right)-i\sin\left(\frac{\pi}{9}\right)\right)^5}$

(b) In a quadratic equation $z^2+(p+iq)z+3i=0$. p and q are real constants. Given that the sum of the squares of the roots is 8. Find all possible pairs of values of p and q.

8. (a) How many different arrangements of letters can be made by using all the letters in the word contact? In how many of these arrangements are the vowels separated?
(b) In how many ways can a team of eleven be picked from fifteen possible players.

9. (a) If α and β are the roots of the equation $x^2-px+q=0$, form the equation whose roots are $\frac{\alpha}{\beta^2}$ and $\frac{\beta}{\alpha^2}$.

(b) If α and β are the roots of the equation $x^2+bx+c=0$, form the equation whose roots are $\frac{1}{\beta^3}$ and $\frac{1}{\alpha^3}$. If in the equation above $\alpha\beta^2=1$, prove that $a^3+c^3+abc=0$

10. (a) If $z=x+iy$ and $\bar{z}$ is the conjugate of z, find the values of x and y such that $\frac{1}{z}+\frac{2}{\bar{z}}=1+i$

(b) If x, y, a and b are real numbers and if $x+iy=\frac{a}{b+\cos\theta+i\sin\theta}$. Show that

$(b^2-1)(x^2+y^2)+a^2=2abx$.

(c) If n is an integer and $z=cos\theta+sin\theta$, show that $2\cos n\theta=z^n+\frac{1}{z^n}$, $2i\sin n\theta=z^n-\frac{1}{z^n}$.

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Use the result to establish the formula $8cos^4\theta = cos4\theta + 4cos2\theta + 3$.

(e) If z is a complex number and $\left|\frac{z-1}{z+1}\right| = 2$, find the equation of the curve in the Argand diagram on which the point representing z lies.

TRIGONOMETRY

11. If $sin\theta + sin\beta = a$ and $cos\theta + cos\beta = b$, show that $\cos^2\left(\frac{\theta-\beta}{2}\right) = \frac{1}{4}(a^2+b^2)$

12. Show that $\sin 7x + \sin x - 2\sin 2x \cos 3x = 4\cos^3 3x$

13. If A, B and C are angles of a triangle, show that:
 (i) $\cos A + \cos(B-C) = 2\sin B \sin C$
 (ii) $\cos\frac{C}{2} + \sin\frac{A-B}{2} = 2\sin\frac{A}{2}\cos\frac{B}{2}$

14. Express $y = 8\cos x + 6\sin x$ in form of $R\cos(x-\alpha)$ where R is positive and α is acute. Hence find the maximum and minimum values of $\frac{1}{8\cos x + 6\sin x + 15}$ and the corresponding angle respectively.

15. Show that:
 (a) $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$
 (b) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$
 (c) Find x if $\tan^{-1} x + \tan^{-1}(1-x) = \tan^{-1}\left(\frac{4}{3}\right)$

16. (a) Show that $\cos 4\theta = \frac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1}$
 (b) Solve the equation $8\cos^4 x - 10\cos^2 x + 2 = 0$ for x in the range of $0° \le x \le 180°$

17. (a) If $\tan\theta = \frac{1}{p}$ and $\tan\beta = \frac{1}{q}$ and $pq = 2$p, show that $\tan(\theta+\beta) = p+q$
 (b) Show that $\sin 2A + \cos 2A = \frac{(1+\tan A)^2 - 2\tan^2 A}{1+\tan^2 A}$

18. If α, β and γ are all greater than $\frac{\pi}{2}$ and less than 2π and $\sin\alpha = \frac{1}{2}$, $\tan\beta = \sqrt{3}$, $\cos\gamma = \frac{1}{\sqrt{2}}$. Find the value of $\tan(\alpha+\beta+\gamma)$ in surd form.

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19. Solve for x in the range 0° to 360°

(a) $3\cos^2 x - 3\sin x\cos x + 2\sin^2 x = 1$

(b) $4\cos x = 3\tan x + 3\sec x$

20. Prove that $4\cos\theta\cos 3\theta + 1 = \frac{\sin 5\theta}{\sin\theta}$. Hence find all the values of θ in the range 0° to 180° for which $\cos\theta\cos 3\theta = \frac{-1}{2}$

VECTORS

21. The coordinates of the points A and B are (0,2,5) and (-1,3,1) and the equation of the line L is $\frac{x-3}{2} = \frac{y-2}{-2} = \frac{z-2}{-1}$

(i) Find the equation of the plane containing the point A and perpendicular to L and verify that B lies in the plane.

(ii) Show that the point C in which L meets the plane is (1,4,3) and find the angle between CA and CB

22. (a) A body moves such that its position is given by $OP = (3sint)i + (3cost)j$ where O is the origin and t is the time. Prove that the velocity of the particle when at P is perpendicular to OP.

(b) The lines L_1 and L_2 have Cartesian equations $\frac{x}{1} = \frac{y+2}{2} = \frac{z-5}{-1}$ and $\frac{x-1}{-1} = \frac{y+3}{-3} = \frac{z-6}{1}$.

Show that L_1 and L_2 intersect and find the coordinates of the point of intersection.

23. (a) Find the acute angle between the lines whose equations are $\frac{x-2}{-4} = \frac{y-3}{3} = \frac{z+1}{-1}$ and $\frac{x-3}{2} = \frac{y-1}{6} = \frac{z+1}{-5}$.

(b) The points A and B have coordinates (1,2,3) and (4,6,-2) respectively and the plane has equation $x + y - z = 24$. Determine the equation of the line AB, hence the angle this line makes with the plane.

24. (a) Find the perpendicular distance of the line $\frac{x-5}{1} = \frac{y-6}{2} = \frac{z-3}{4}$ from the point (-6,-4,-5).

(b) Find the shortest distance between the two skew lines $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x}{2} = \frac{y+1}{1} = \frac{z-1}{3}$ respectively.

(c) Find the perpendicular distance of the plane $2x - 14z + 5z = 10$ from the origin.

25. (a) Show that the line $\frac{x-2}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ is parallel to the plane $4x - y - 3z = 4$ and find the perpendicular distance from the line to the plane.

(b) Find the Cartesian equation of the line of intersection of the two planes $2x - 3y - z = 1$ and $3x + 4y + 2z = 3$.

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26. (a) Find the Cartesian equation of the plane containing the point (1,3,1) and parallel to the vectors $\begin{pmatrix}1\\-1\\3\end{pmatrix}$ and $\begin{pmatrix}2\\1\\-3\end{pmatrix}$

(b) Find the Cartesian equation of the plane containing the points (1,2,-1), (2,1,2) and (3,-3,3).

27. Given the points A, B and C with coordinates (2,5,-1), (3,-4,2) and C(-1,2,1). Show that ABC is a triangle and find the area of the triangle ABC

28. (a) Find the angle between the parallel planes $3x + 2y - z = -4$ and $6x + 4y - 2z = 6$.
(b) Find the acute angle between the planes $2x + y + 3z = 5$ and $2x + 3y + z = 7$

29. The points A and B have coordinates (2,1,1) and (0,5,3) respectively. Find the equation of the line AB. If C is the point (5,-4,2). Find the coordinates of D on AB such that CD is perpendicular to AB. Find the equation of the plane containing AB and perpendicular to the line CD.

30. (a) Given that $OP = \begin{pmatrix}4\\-3\\5\end{pmatrix}$ and $OQ = \begin{pmatrix}1\\0\\2\end{pmatrix}$, find the coordinates of the point R such that $\overline{PR} = \overline{PQ} = 1:2$ and the points P, Q and R are collinear.
(b) A and B are the points (3,1,1) and (5,2,3) respectively, and C is a point on the line $\boldsymbol{r} = \begin{pmatrix}2\\4\\-2\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$. If angle BAC=$90^0$, find the coordinates of C

ANALYSIS

31. Differentiate from first principles
(a) $y = \tan^{-1} x$
(b) $y = ax^n$
(c) $y = \sin 3x$

32. Find the derivative of:
(a) $y = 5\sin^{-1}(4x)$
(b) $y = \tan^{-1}\left(\dfrac{1+\tan x}{1-\tan x}\right)$
(c) $y = \dfrac{\sin x}{x^2 + \cos x}$
(d) $y = \sqrt{\dfrac{x}{1+x}}$

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33. Find:

(a) $\int \sin^{-1} x$

(b) $\int \frac{dx}{x^2+4x+13}$

(c) $\int \frac{dx}{x\log_e x}$

(d) $\int \frac{dx}{(1+x^2)\tan^{-1} x}$

(e) Show that $\int_0^2 \sqrt{\frac{x}{4-x}}dx = \pi - 2$

(f) Show that $\int_1^{10} x\log_{10} x = 50 - \frac{99}{4\ln 10}$

34. (a) If $x = t^3$ and $y = 2t^2$. Find $\frac{dy}{dx}$ in terms of t and show that when $\frac{dy}{dx} = 1$, $x = 2$ or $x = \frac{10}{27}$

(b) If $y = \frac{2t}{1+t^2}$ and $x = \frac{1-t^2}{1+t^2}$, find $\frac{d^2y}{dx^2}$ in terms of t

35. Given that:

(a) $y = \sqrt{4+3\sin x}$, show that $2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 4$

(b) $y = e^{2x}\cos 3x$, show that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

(c) $y = \left(x+\sqrt{1+x^2}\right)^p$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - p^2y = 0$

(d) $y = \sin(\log_e x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

36. (a) Find the volume generated when the area enclosed by the curve $y = 4x - x^2$ and the line $y = 2x$ is rotated completely about the x – axis.

(b) Find the area contained between the two parabolas $4y = x^2$ and $4x = y^2$.

(c) Find the area between the curve $y = x^3$, the x – axis and the lines $y = 1, y = 8$.

(d) Find the area of the curve $x^2 + 3xy + 3y^2 = 1$

(e) Show that in the solid generated by the revolution of the rectangular hyperbola $x^2 - y^2 = a^2$ about the x – axis, the volume of the segment of height a from the vertex is $\frac{4}{3}\pi a^3$

37. (a) A right circular cone of semi – vertical angle θ is circumscribed about a sphere of radius R. show that the volume of the cone is $V = \frac{1}{3}\pi R^3(1+\operatorname{cosec}\theta)^3\tan^2\theta$ and find the value of θ when the volume is minimum.

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(b) Water is poured into a vessel, in the shape of a right circular cone of vertical angle 90°, with the axis vertical, at the rate of $125\text{cm}^3/s$. At what rate is the water surface rising when the depth of the water is 10cm?

38. Sketch the curve $y = \frac{x}{x+2}$. Find the area enclosed by the curve, the lines $x = 0, x = 1$ and the line $y = 1$. Also find the volume generated when this area revolves through 2π radians about the line $y = 1$.

39. Solve the differential equations below:

(a) $\frac{1}{3x}\frac{dy}{dx} + \cos^2 y = 1$,when $x = 2$ and $y = \frac{\pi}{4}$

(b) $(x-y)\frac{dy}{dx} = x+y$, when $x = 4$ and $y = \pi$

(c) $\frac{dy}{dx} + 3y = e^{2x}$, when $x = 0$ and $y = \frac{6}{5}$

40. In a certain type of chemical reaction a substance A is continuously transformed into a substance B. throughout the reaction, the sum of the masses of A and B remains constant and equal to m. The mass of B present at time t after the commencement of the reaction is denoted by x. At any instant, the rate of increase of mass of B is k times the mass of A where k is a positive constant.

(a) Write down a differential equation relating x and t

(b) Solve this differential equation given that $x = 0$ and $t = 0$. Given also that $x = \frac{1}{2}m$ when $t = \ln 2$, determine the value of k and show that at time t, $x = m(1 - e^{-t})$. Hence find:

(i) The value of x (in terms of m) when $t = 3\ln 2$

(ii) The value of t when $x = \frac{3}{4}m$

GEOMETRY

41. (a) Find the equation of a line which makes an angle of 150° with the x – axis and y – intercept of -3 units.

(b) Find the acute angle between the lines $3y - x = 4$ and $6y - 3x - 5 = 0$

(c) OA and OB are equal sides of an isosceles triangle lying in the first quadrant. OA and OB make angles θ_1 and θ_2 with x – axis respectively. Show that the gradient of the bisector of the acute angle AOB is $\operatorname{cosec}\theta - \cot\theta$ where $\theta = \theta_1 + \theta_2$

(d) Find the length of the perpendicular from the point P(2,-4) to the line $3x + 2y - 5 = 0$

42. (a) Find the equation of the circle with centre (4,-7) which touches the line $3x + 4y - 9 = 0$

(b) Find the equation of the circle through the points (6,1), (3,2), (2,3)

(c) Find the equation of the circumcircle of the triangle formed by three lines $2y - 9x + 26 = 0$, $9y + 2x + 32 = 0$ and $11y - 7x - 27 = 0$

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43. (a) Find the length of the tangent from the point (5,6) to the circle $x^2+y^2+2x+4y-21=0$.

(b) Find the equations of the tangents to the circle $x^2+y^2=289$ which are parallel to the line $8x-15y=0$

(c) Find the equation of the circle of radius $12\frac{4}{5}$ which touches both the lines $4x-3y=0$ and $3x+4y-13=0$ and intersects the positive y – axis.

(d) A circle touches both the x – axis and the line $4x-3y+4=0$. Its centre is in the first quadrant and lies on the line $x-y-1=0$. Prove that its equation is $x^2+y^2-6x-4y+9=0$

44. Find the equations of the parabolas with the following foci and directrices:

(i) Focus (2,1), directrix $x=-3$

(ii) Focus (0,0), directrix $x+y=4$

(iii) Focus (-2,-3), directrix $3x+4y-3=0$

45. (a) Show that the curve $x=5-6y+y^2$ represents a parabola. Find its focus and directrix, hence sketch it.

(b) Find the equation of the normal to the curve $y^2=4bx$ at the point $P(bp^2,2bp)$. Given that the normal meets the curve again at $Q(bq^2,2bq)$, prove that $p^2+pq+2=0$

46. (a) Show that the equation of the normal with gradient m to the parabola $y^2=4ax$ is given by $y=mx-2am-am^3$.

(b) P and Q are two points on the parabola $y^2=4ax$ whose coordinates are $P(ap^2,2ap)$ and $Q(aq^2,2aq)$ respectively. If OP is perpendicular to OQ, show that $pq=-4$ and that the tangents to the curve at P and Q meet on the line $x+4a=0$

47. (a) A conic is given by $x=4\cos\theta$, $y=3\sin\theta$. Show that the conic is an ellipse and determine its eccentricity

(b) Given that the line $y=mx+c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $c^2=a^2m^2+b^2$. Hence determine the equations of the tangents at the point (-3,3) to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

48. (a) Show that the locus of the point of intersection of the tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which are at right angles to one another is a circle $x^2+y^2=a^2+b^2$.

(b) The normal to the ellipse $x^2+y^2=100$ at the points A(6,4) and B(8,3) meet at N. If P is the mid – point of AB and O is the origin, show that OP is perpendicular to ON.

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49. (a) P is a point $(ap^2, 2ap)$ and Q the point $(aq^2, 2aq)$ on the parabola $y^2 = 4ax$. The tangents at P and Q intersect at R. Show that the area of triangle PQR is $\frac{1}{2}a^2(p-q)^3$

(b) The normal to the parabola $y^2 = 4ax$ at $P(ap^2, 2ap)$ meets the axis of the parabola at M and MP is produced beyond P to Q so that MP = PQ. Show that the locus of Q is $y^2 = 16a(x+2a)$

50. (a) The normal to the rectangular hyperbola $xy = 8$ at the point (4,2) meets the asymptotes at M and N. Find the length of MN

(b) The tangent at P to the rectangular hyperbola $xy = c^2$ meets the lines $x - y = 0$ and $x + y = 0$ at A and B and Δ denotes the area of triangle OAB where O is the origin. The normal at P meets the x – axis at C and the y – axis at D. if Δ_1 denotes the area of the triangle ODC. Show that $\Delta^2\Delta_1 = 8c^6$

END

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APPLIED MATHEMATICS REVISION QUESTIONS

STATISTICS.

1. Consider the following data

CI	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 - 39
F	2	3	6	2	4	1

i) Draw the histogram of the data

ii) Determine the mode from your histogram or otherwise

iii) Calculate the mean distribution using the assumed formula

2. The table below shows the marks obtained by students of mathematics in a certain school.

Marks	Number of students
30 - < 40	2
40 - < 50	15
50 - < 55	10
55 - < 60	11
60 - < 70	30
70 - < 90	29
90 - < 100	3

i) Calculate the mean, median and standard deviation

ii) Draw a histogram for the data, hence determine the modal mark

iii) Draw an ogive, hence determine the median from the ogive and compare it with the calculated value.

3. Consider the following data below

137	140	150	140	157	131
141	142	162	169	166	129
138	170	170	152	161	139
122	131	139	170	165	145
140	147	125	134	153	145
151	128	129	121	154	167
122	165	128	149	140	136
130	170	133	139	136	150

Construct a frequency table with classes of equal width starting with 121 – 130 and use it to calculate the mean, mode and standard deviation

4. Consider the following data

CI	10 – 14	15 – 19	20 – 29	30 – 34	35 – 44	45 - 49
f	2	3	6	2	4	1

i) Draw the histogram of the data

ii) Determine the mode from your histogram

iii) Calculate the quartile deviation

5. a) In 1995 the prices of commodities A, B and C were shs. 720, shs. 830 and shs. 950 respectively. Given that the prices after 5 years were shs. 860, shs. 940 and shs. x and the simple aggregate price index number was 140. Find x

b) The price of a radio in 1995 was shs. 30,000. The index number for the price of a radio in 1985 was 1.6 based on 1975. 1n 1995, it was 0.75 based on 1985. Calculate

i) The prices of the radio in 1975 and 1985

ii) The price relative in 1995 based on 1975

6. A pharmacist had the following record of unit price and quantities of drugs sold for the years 1990 and 1991

Drug	Quantities in cartons		Unit price per carton	
	1990	1991	1990	1991
Aspirin	40	45	80	125
Panadol	70	90	100	90
Quinine	08	10	55	70
flue caps	10	10	90	100

Using 1990 as the base year, calculate

i) The price relative for each drug

ii) The simple aggregate price index number for 1991

iii) Fisher's index number for 1991

7. The table below shows the performance of 100 students in a resource examination

Score (%)	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49	50 - 59
Candidates	10	x	25	30	y	10

i) Given the median is 30.5, determine the values of x and y

ii) Hence calculate the average score and the mode

iii) Construct a cumulative frequency curve and use it to estimate the inter-quartile range

8. Using the figures in the table below

Food	2000		2005	
	Quantity in kg	Price per kg(shs)	Quantity in kg	Price per kg(shs)
Maize	20	650	25	700
Wheat	10	1500	8	1600
Beans	5	150	8	200

Calculate

i) Paasche aggregate price index

ii) Laspeyre aggregate price index

9. The cost of making a cake is calculated from the baking flour, sugar, milk and eggs. The table below gives the cost of these items in 1990 and 2000

Item	1990	2000	Weight, w
Flour per kg	600	780	12
Sugar per kg	500	400	5
Milk per litre	250	300	2
Eggs per egg	100	150	1

Using 1990 as the base year, calculate

i) The price relative for each item. Hence find simple price index for the cost of making a cake

ii) The simple aggregate price index number for 2000

iii) Fisher's index number for 2000

10. The following items are used in 1990 and 1985 as shown in the table below

Item	1985 price (£)	1990 price (£)	Weight, w
Radio	56	60	4
Shoes	45	50	2
Cap	15	20	1

Calculate the weighted price index number using 1985 as a base year

11. Given that A and B are mutually exclusive events such that P(A) = 0.3 and P(AUB) = 0.7. Find

i) P(B) ii) $P(A^1 \cap B)$

iii) $P(A^1 U B)$ iv) $P(A^1 \cap B^1)$

12. Given two events A and B such that $P(A^1) = {}^2/_3$, $P(B) = {}^1/_2$, and $P(A \cap B) = {}^1/_{12}$. Find

i) P(A) ii) P(AUB)

iii) $P(A^1 \cap B)$ iv) $P(A^1 U B^1)$

13. Events A and B are such that $P(A) = {}^1/_2$, $P(B) = {}^3/_8$, and $P({}^A/_B) = {}^7/_{12}$. Find

i) $P(A \cap B)$ ii) $P({}^B/_A)$

iii) $P(A^1 \cap B)$ iv) $P\left({}^B/_{A^1}\right)$

14. Two independent events A and B are such that P(A) = 0.4, P(B) = b and P(AUB) = 0.7. Find

i) The value of b ii) $P(A \cap B)$

iii) $P(A \cap B^1)$ iv) $P(A \text{ U } B^1)$

15. There are 3 black and 2 white balls in each of two bags. A ball is taken from the first bag and put in the second bag, and then a ball is taken from the second bag into the first bag. What is the probability that there are now the same numbers of black and white balls in each bag as there were to begin with?

16. A fair coin is tossed four times. Use the tree diagram to calculate the probability of obtaining

i) Exactly three heads

ii) At least two tails

iii) Exactly one head

iv) Not more than three tails

17. The probabilities that a man makes a journey by car, air and road are respectively $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$. If the probabilities of an accident occurring when he uses these means of transport are $\frac{1}{3}, \frac{3}{10}$ and $\frac{1}{10}$ respectively

i) What is the probability of an accident occurring

ii) If the accident is known to have happened, find the probability that the man was travelling by Air

ii) If it is known that the man reached safely, find the probability that he used the road

18. Three workers of dairy cooperation, Joy, Jane and Juliet seal milk sackets. On a particular day Joy seals 48%, Jane 30% and Juliet 22%, the probability that Joy seals wrongly is 0.53, Jane seals wrongly is 0.30 and Juliet seals wrongly is 0.17. Find the probability that a sacket was sealed wrongly and a wrongly sealed sacket found by the checker was sealed by Joy.

19. A bag contains white, yellow and blue beads in the ratio of 3: 4: 1. Two beads are selected at random without replacement, one after the other, obtain the probability that

i) Two of the selected beads are of the same colour

ii) The selected beads of different colours

20. A committee of 5 is selected from 4 men and 3 women.

i) In how many ways can this be done if there must be more men than women?

ii) What is the probability that the committee consists of 2 men

iii) A woman is selected at random, find the probability that she belongs to this committee.

21. A discrete random variable X has a pdf, f(x) given by

$$f(x) = \begin{cases} k(x+1), & x = 1, 2, 3, 4. \\ kx, & x = 5, 6, 7 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) $P(2 \leq x < 7)$

iii) The mean and mode of X

iv) The standard deviation of X

v) The semi inter-quartile range

22. A random variable X has the following cdf, F(x) given by

$$F(x) = \begin{cases} \frac{kx}{4}, & x = 3, 4, 5 \\ k(x-3), & x = 6, 7, 8 \\ 1, & x \geq 8 \end{cases}$$

Find

i) The value of the constant, k

ii) The pmf of x

iii) Expectation of X

iv) $P(x \geq E(x))$

v) Median and variance of X

23. The number of days the machine breaks down in a week follows a discrete random variable X with the following pdf, f(x) given by

$$f(x) = \begin{cases} kx^2, & x = 1, \dots, 4. \\ k(7-x)^2, & x = 5, 6 \\ \frac{k}{7}, & x = 7 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) The mean number of days the machine breaks down in a week

iii) The probability that the machine breaks down not more than 3 days in a week

24. A discrete random variable X has a probability density function, f(x) given by

$$f(x) = \begin{cases} \frac{x}{k}, & x = 1, 2, \dots, n \\ 0, & \text{elsewhere} \end{cases}$$

Given that the expectation of X is 3. Find

i) The value of n and the constant, k   ii) The median of X

iii) The variance of X   iv) P(x = 2/x ≥2)

v) The cdf of X

25. The number of days the machine breaks down in a month follows a discrete random variable X with the following pdf, f(x) given by

$$f(x) = \begin{cases} k\left(\frac{1}{4}\right)^x, & x = 0, 1, 2, \dots \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) The probability that the machine breaks down not more than 2 times in a month

26. a) The probability distribution of a discrete random variable X is given by

$$P(X = r) = \begin{cases} kr, & r = 1, 2, 3, \dots, n \\ 0, & \text{elsewhere} \end{cases}$$

i) Show that the constant, $k = \frac{2}{n(n+1)}$

ii) Find in terms of n the mean and variance of X

b) The discrete random variable X has a probability density function given by

$$f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, 3, 4, 5 \\ C, & x = 6 \\ 0, & \text{otherwise} \end{cases}$$

Determine

i) The value of the constant, C

ii) The mean of X

iii) The mode of X

27. A random variable X has the following probability distribution

$P(X = -2) = P(X = -1) = 2P(X= 0)$, $P(X = 1) = 0.2$, $2P(X = 2) = P(X =3)$

The mean of X equals the probability with which X assumes the value of -1. Find

i) $P(X = 2)$ and $P(X = 0)$

ii) $P(x \leq 2/x \geq 0)$

iii) The standard deviation of X

iv) The upper quartile

28. The random variable X takes integer values only and has a pdf given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5 \\ k(10 - x), & x = 6, 7, 8, 9 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of the constant, k

ii) $E(X)$ and $Var(X)$

iii) $E(2X - 3)$ and $Var(2X - 3)$

iv) $P(2 \leq 2X - 2 \leq 11)$

29. Two random variables X and Y take on integer values with probabilities as given below

X	2	3	4	5	6
P(X = x)	0.25	0.15	0.10	0.30	0.20

Y	-1	0	1	2	3
P(Y = y)	0.20	0.25	0.40	0.10	0.05

Find;

i) $E(2X - 4)$

ii) $Var(2X + Y)$

iii) $Var(3X - 4Y)$

iv) $Var(X + Y)$

30. A discrete random variable X has the following distribution given by

X	1	2	3	4	5
P(X = x)	$\frac{4}{15}$	$\frac{3}{40}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{11}{24}$

Two other random variables Y and Z are defined in terms of X as follows

Y = 2X + 1 and Z = 3X – 2

Find

i) The pmf of Y and Z   ii) Standard deviation of Z

iii) The variance of Y   iv) Draw the graph of cdf of Z

v) $P(3 < Z < 9)$

vi) Plot a graph of pmf of Y use it to find the mode

31. A random variable X has pdf, f(x) where

$$f(x) = \begin{cases} k[2 - (x+1)^2], & 1 \leq x \leq 3 \\ 0, & \text{elsehere} \end{cases}$$

Determine the

i) The value of a constant

ii) The c.d.f, F(x) of X

32. A random variable X has p.d.f f(x) given by

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 2 \\ 2k(x-1)^2, & 2 \leq x \leq 5 \\ 0, & \text{elesewhere} \end{cases}$$

Determine the

i) Value of a constant, k   ii) mean of X

iii) Standard deviation of X   iv) $P(|x - 3| < 1)$

33. A random variable X has pdf f(x) given by

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 1 \\ \frac{k}{2}(x+1), & 1 \leq x \leq 3 \\ 2k, & 3 \leq x \leq 4 \\ 0, & \text{elesewhere} \end{cases}$$

i) Sketch a graph of f(x) and use it to find   ii) The constant, k

iii) Calculate the cdf, F(x) of X   iv) $P(1 \leq x \leq 4)$

v) Calculate the mean and standard deviation of x

34. A random variable X has the following cdf, F(x) given by

$$F(x) = \begin{cases} 0, & x \le 0 \\ \frac{k}{2}x^2, & 0 \le x \le 1 \\ \frac{k}{2} + \frac{k}{4}(6x - x^2 - 5), & 1 \le x \le 3 \\ 1, & x \ge 3 \end{cases}$$

Find the

i) Value of the constant, k ii) The pdf, f(x) of X

iii) $P(1 \le 2x \le 3)$ iv) Sketch the graph of f(x)

v) $P(0 \le x \le 2/0.5 \ge x)$

35. A random variable X has probability density function, f(x) given by

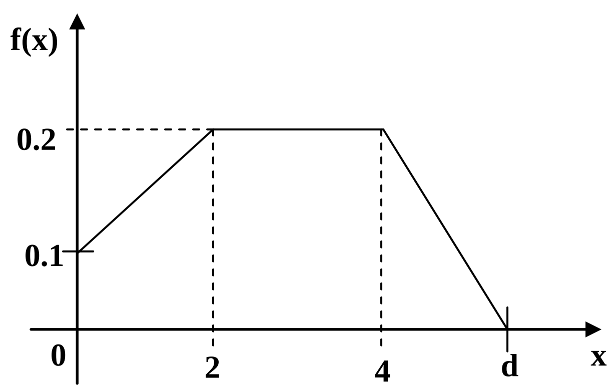
$$f(x) = \begin{cases} \frac{2}{3}a(x + a), & -a \le x \le 0 \\ \frac{1}{3a}(2a - x), & 0 \le x \le 2a \\ 0, & \text{elesewhere} \end{cases}$$

Determine the

i) Value of the constant, a ii) median of X iii) $P(x \le 1.5/x > 0)$

iv) Cumulative distribution function, F(x) and sketch it

36. The motion of a motorist may be modeled into a continuous random variable X with the graph of f(x) as shown below;



Calculate the

i) value of d ii) pdf, f(x) of X

iii) standard deviation iv) cdf, F(x) of X

v) the median of X

37. A continuous random variable X has a pdf given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k, & 1 \le x \le 2 \\ 2(3-x), & 2 \le x \le 3 \\ 0, & \text{elesewhere} \end{cases}$$

i) Sketch f(x) and use it to find the value of k

ii) Calculate the mean of X iii) Find $P(x < 2.5)$

iv) Determine the cumulative distribution function, F(x)

38. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} \frac{1}{3}(x-2), & 2 \le x \le 3 \\ a, & 3 \le x \le 5 \\ 2-bx, & 5 \le x \le 6 \\ 0, & \text{elesewhere} \end{cases}$$

i) Find the value of a and b

ii) Determine $P(x > \frac{11}{2})$

iii) Determine the inter-quartile range

iv) Obtain the expression for F(x)

39. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} kx(3-x), & 0 \le x \le 2 \\ k(4-x), & 2 \le x \le 4 \\ 0, & \text{elesewhere} \end{cases}$$

Find

i) The value of k iii) The mean of X

ii) $P(1 < x < 3)$

iii) Cumulative distribution function, F(x)

40. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k(4-x^2), & 1 \le x \le 2 \\ 0, & \text{elesewhere} \end{cases}$$

Find

i) The value of k

ii) E(X) and Var (x)

iii) Cumulative distribution function, F(x)

41. A continuous rectangular random variable X has limits 3 and 8

i) Draw the graph of its pdf

ii) Use the graph to find $P(x > 5)$

iii) Find the cdf and draw its graph

iv) Find the median of X

42. The continuous random variable X has the pdf, f(x) given by

$$f(x) = \begin{cases} \frac{1}{k}, & 24 \le x \le 34 \\ 0, & Otherwise \end{cases}$$

Find the

i) Value of k

ii) Mean, median and quartile deviation

iii) Cdf of X

iv) Value of a if $P(x > a) = 0.65$

43. It is known that a continuous random variable X has uniform distribution over the interval [20, 30]. Find the

i) Mean and standard deviation of X

ii) Median and quartile deviation of X

iii) $P(25 < x < 29/x > 27)$

iv) cdf of X

44. A continuous random variable X has uniform distribution over the interval [d, e]. d < e. Show that

i) The mean is equal to the median

ii) $SD(x) = \frac{e-d}{2\sqrt{3}}$

iii) If $E(x) = 3$ and $Var(x) = \frac{4}{3}$ then $e = 5$ and $d = 1$

45. A discrete random variable X has uniform distribution over the interval [a, b].

a < b, a and b are integers

i) Show that its pdf satisfies properties of the pdf of a discrete random variable

ii) Prove that its mean is ½ (a + b + 1)

iii) Obtain the expression for the variance hence its SD

iv) Find its median and quartile deviation if a = 10 and b = 25

MECHANICS

1. A train approaching a station covers successive half kilometers in 16s and 20s respectively. Assuming the retardation to be uniform, find the further distance the train run before coming to a stop.
2. Two points P and Q are x m apart. A boy starts from rest at P and moves directly towards Q with an acceleration of a ms^{-2} until he acquires a speed of Vms^{-1}. He maintains this speed for T seconds and then brought to rest at Q under a retardation of a ms^{-2}. Prove that $T = \frac{x}{V} - \frac{V}{a}$
3. A car started from rest accelerated uniformly for 2 minutes and then maintained a speed of $50kmh^{-1}$. Another car started 2 minutes later from the same spot and this car too accelerated uniformly for 2 minutes and it then maintained a speed of $75kmh^{-1}$.

   i) Draw a velocity – time graph and find when and where the second car overtook the first.

   ii) The car maintained the speed of $50kmh^{-1}$ for 10minutes. It then decelerated uniformly for further 2½ minutes before coming to rest. How far has the car travelled from the start?
4. A motor car A passes a certain point P with a speed of $7.5ms^{-1}$ and an acceleration of $0.3ms^{-2}$. Five seconds later a car B passes P with a speed of $15.0ms^{-1}$ and an acceleration of $0.2ms^{-2}$. Prove that if their maximum speed is $30.0ms^{-1}$, B will ultimately be travelling 131m a head of A.
5. A car starting from rest is uniformly accelerated during the first 0.5km of its run, then run 1.5km at a uniform speed and is afterwards brought to rest in ¼km under uniform retardation. If the time for the whole journey is 5minutes, find the uniform acceleration and uniform retardation.
6. A particle starts from rest and moves in a straight line with uniform acceleration. In 4 seconds of its motion it travels 12m and in the next 5 seconds it travels 30m. Find its

   i) Acceleration

   ii) Final velocity

7. A train travels a long a straight track between two stations A and B. the train starts from rest at A and accelerates at $1.25ms^{-2}$ until it reaches a speed $20ms^{-1}$. It then travels at this speed for a distance of 1.56km and then decelerates at $2ms^{-2}$ to come rest at B.
   i) Sketch a velocity-time graph for the motion of the train
   ii) Find the distance from A to B
   iii) Find the total time of the journey

8. A, B and C are three points which lie in that order on a straight road with AB = 95m and BC = 80m. a car is travelling along the road in the direction ABC with constant acceleration of ams^{-2}. The car passes through A with speed u ms^{-1} reaches B later and C two seconds after that. Find the values of a and u.

9. Two stations A and B are a distance of 6x mtres apart along a straight line. A train starts from rest at A and accelerates uniformly to a speed of Vms^{-1} covering a distance of x metres. The train then maintains this speed until it has travelled a further 3x metres. It then retards uniformly to rest at B. sketch a velocity- time graph for the motion of the car and show that if T is the time taken to travel from A to B then T = $\frac{9x}{V}$ seconds.

10. P, Q and R are points on a straight road such that PQ = 20m and QR = 55m. a cyclist moving with uniform acceleration passes P and then notices that it takes him 10s and 15s to travel between (P and Q) and (Q and R) respectively. Find his uniform acceleration.

11. A body is projected vertically upwards with a velocity of $25ms^{-1}$. Find
    i) How high it will go
    ii) What times elapses before it is at a height of 20m
    iii) The time to reach maximum height hence time of flight

12. A ball is thrown vertically upwards with a speed of $42ms^{-1}$. If it falls past the point of projection into a sea of depth 80m, find when it strikes the bottom of the sea.

13. A particle is projected from a point O with an initial velocity 3i + 4j. Find in vector form the velocity and position of the particle at any time, t.

14. A stone is thrown vertically upwards with a velocity of $25ms^{-1}$, if another stone is thrown vertically upwards 4 seconds later with the same speed from the same point of projection. Determine when and where the two stones meet.

15. A stone is dropped from the top of the building and at the same instant another stone is thrown vertically upwards from the bottom of the building with a speed of $20ms^{-1}$. They pass each other three seconds later. Find the height of the building.

16. A particle is projected vertically upwards with a certain velocity and it is found that when it is 400m from the ground it takes 8 seconds to return to the same point again. Find the velocity of projection and the time of flight.

17. The ball is thrown vertically down wards from the top of the tower and has an initial speed of $4ms^{-1}$. If the ball hits the ground 2 seconds later. Find

    i) the height of the tower

    ii) the speed at which the ball strikes the ground.

18. A stone is thrown vertically upwards from the ground level at a speed of $24.5ms^{-1}$. Find how long after projection the stone is at a height of 19.6m above the ground for the first time and the second time and how long is the stone at least 19.6m above the ground level.

19. A ball is thrown vertically upwards with a speed, u after time, t another ball is thrown vertically upwards from the same point with the same speed. Prove that they will meet after elapse of $\left(\frac{t}{2}+\frac{u}{g}\right)$ seconds from the time the first particle was projected hence show that the distance travelled is $\left(\frac{4u^2-g^2t^2}{8g}\right)$ m.

20. A particle is projected vertically upwards from a point O with a speed $\frac{4}{3}$ V. after it has travelled a distance $\frac{2}{5}$ x above O on its upward motion, a second particle is projected vertically upwards from the same point and with the same initial speed. Given that the particles collide at a height $\frac{2}{5}$ x above O, x and V being constant, show that

i) at maximum height H, $8V^2 = 9gH$

ii) when the particles collide $9x = 20H$

21. Find the angle between a and b, given that

$a = 5i + 6j + k$ and $b = 2i + j$

22. A particle of mass 5kg at rest at appoint (1, -4, 4) is acted upon by three forces $F_1 = 3i + 3j$, $F_2 = 2j + 4k$ and $F_3 = 2i + 6k$. Find

i) Acceleration of the particle

ii) Velocity and speed of the particle after 4 seconds

iii) Position and the distance of the particle after 4 seconds

23. The forces $F_1 = 2i + 3j$, $F_2 = i + 3j$ and $F_3 = i + 2j$ act on a particle of mass 2kg located at (1, -1). Find

i) The magnitude and direction of the resultant force

ii) Position and the distance of the particle after 4 seconds

24. Three forces $F_1 = 6i + 3j$, $F_2 = -2i + 3j$ and $F_3 = \lambda i$ act on a particle at the origin, if the magnitude of the resultant is 10N. Calculate the two possible values of λ and the two possible directions of the line of action of the resultant.

25. A particle of mass 3kg moving on the curve described by

$r = 4\sin 3t i + 8\cos 3t j$ where r is the position vector at time, t.

i) Determine the position and the velocity of the particle at time, t = 0s

ii) Show that the force acting on the particle is –27r.

26. A particle of mass 2kg initially at rest at (0, 0, 0) is acted upon by the force $\begin{pmatrix} 2t \\ t \\ 3t \end{pmatrix}$ N. Find

i) Acceleration and velocity after 3 seconds.

ii) Distance moved after 3 seconds.

27. A particle of weight 8N is attached at a point B of a light inextensible string AB. It hangs in equilibrium with point A fixed and AB at an angle of 30^o to the vertical. A force, F at B acting at right angles to AB keeps the particle in equilibrium. Find the magnitude of F and the tension in the string.

28. A particle of mass 3kg is attached to the lower end B of an inextensible string. The upper end A of the string is fixed to a point on the ceiling of a roof. A horizontal force of 22N and an upward vertical force of 4.9N act upon the particle making it be in equilibrium with the string making an angle, $\propto$ with the vertical. Find the value of $\propto$ and the tension in the string.

29. A non-uniform rod of mass 9kg rests horizontally in equilibrium supported by two light inextensible strings tied to the ends of the rod. The strings make angles of 50^o and 60^o with the rod. Calculate the tensions in the strings.

30. A particle moving with an acceleration given by

$a = 4e^{-3t}i + 12\sin tj - 7\cos tk$ is located at a point (5, -6, 2) and has velocity $v = 11i - 8j + 3k$ at time $t = 0$. Find the

i) Magnitude of the acceleration when $t = 0$

ii) Velocity at any time, t

iii) Displacement at any time, t

31. A particle with position vector $10i + 3j + 5j$ moves with constant speed of $6ms^{-1}$ in the direction $i + 2j + 2k$. Find its distance from the origin after 5 seconds.

32. A particle of mass 2 units moves under the action of force which depends on time, t given by $F = 24t^2i + (36t - 16)j$. given that $t = 0$, the particle is located at $3i - j$ and has a velocity $6i + 15j$. Find

i) the kinetic energy of the particle at $t = 2$

ii) the impulse in moving the particle from $t = 1$ to $t = 2$ s

33. If the $a = 6\sin 6ti + 9\cos 3t\, j$. find displacement when $t = \frac{1}{3}$ given that $v = i + 3j$ and $s = 5i + 2j$ when $t = \frac{\pi}{6}$

34. An object of mass 5kg is initially at rest at a point position vector is $-2i + j$. if it is acted upon by a force $F = 2i + 3j - 4k$. Find

i) the acceleration

ii) the velocity after 3 seconds

iii) the distance from the origin after 3 seconds.

35. A particle of mass 2m rests on a rough plane inclined at an angle of $\tan^{-1}(3\mu)$ where μ is the coefficient of friction between the particle and the plane. The plane is acted upon by a force of PN

a) Given that the force acts along the line of greatest slope and that the particle is on a point of sliding up. Show that the maximum force possible to maintain the particle in equilibrium is

$$P_{max} = \frac{8\mu mg}{\sqrt{1 + 9\mu^2}}$$

b) Given that the force acts horizontally in a vertical plane through the line of greatest slope and that the particle is on a point of sliding down the plane. Show that the force required to maintain the particle in equilibrium is

$$P = \frac{4\mu mg}{1 + 3\mu^2}$$

36. A carton of mass 3kg rests on a rough plane inclined at an angle of 30^0 to the horizontal. The coefficient of friction between the carton and the plane is $\frac{1}{3}$. Find a horizontal force that should be applied to make the carton just about to move up the plane.

37. A body of mass 8kg rests on a rough plane inclined at θ to the horizontal. If the coefficient of friction is μ, find the least horizontal force in terms of μ, θ and g which will hold the body in equilibrium.

38. A particle of mass 2kg rests on a rough inclined plane at an angle $\sin^{-1}\left(\frac{5}{13}\right)$. A force of 8N acts on the particle along the line of greatest slope
i) given that the particle is about to move up the plane, calculate the coefficient of friction between the particle and the plane
ii) if the 8N force is removed, find the acceleration of the particle down the plane.

39. A box of mass 2kg is at rest on a plane inclined at 25° to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane.

40. A particle of mass ½ kg is released from rest and slides down a rough plane inclined at 30° to the horizontal. It takes 6 seconds to go 3m.
i) find the coefficient of friction between the particle and the plane
ii) what minimum horizontal force is needed to prevent the particle from moving.

41. A particle of weight, W is at rest on an inclined plane under the action of a force, P acting parallel to the line of greatest slope of the plane in an upward direction. The angle of friction between the particle and the plane is λ and the angle of inclination of the plane to the horizontal is 2λ. Show that $P_{max} = W\tan\lambda(4\cos\lambda - 1)$ and $P_{min} = \mu W$ respectively. Calculate the coefficient of friction between the particle and the plane.

42. A particle of mass 2kg rests on a rough horizontal ground. The coefficient of friction between the particle and the ground is ½. Find the magnitude of the force, P acting upwards on the particle at an angle of 30°to the horizontal which will just move the particle.

43. A parcel of mass 2kg is place on a rough plane which is inclined at an angle of 45^{o} to the horizontal. The coefficient of friction between the parcel and the parcel and the plane is 0.25. Find the force that must be applied to the parcel in a direction parallel to the plane so that

i) the parcel is just prevented from sliding down the plane

ii) the parcel is just on the point of moving up the plane

iii) the parcel moves up the plane with an acceleration of $1.5ms^{-2}$

44. when at an angle of elevation, $\propto$ a gun fires a shot to hit a mark P on the horizontal plane, when the angle is reduced to 15^{o} the shot falls 100m short of P but when the elevation is 45^{o} it falls 400m beyond P. find the value of $\propto$ and distance of P from the gun.

45. The horizontal and vertical components of the initial velocity of a particle projected from a point O on the horizontal plane are p and q respectively.

i) Express the vertical distance Y travelled in terms of the horizontal distance X and the components of p and q.

ii) Find the greatest height, H attained and the range, R on the horizontal plane through O. hence show that $Y = \frac{4HX}{R^2}(R - X)$. Given that the particle passes through the point (20, 80) and H = 100m. Find the velocity of projection.

46. A particle P is projected from a point A with an initial velocity of $60ms^{-1}$ at an angle of 30^{o} to the horizontal. At the same time and the same instant a particle Q is projected in opposite direction with initial speed $50ms^{-1}$ from a point at the same level with A and 100m from A. given that the particles collide, find

i) the angle of projection of q

ii) the time when collision occurs

47. If T is the time of flight and x the horizontal range of a projectile, prove that $gT^2 = 2x \tan \propto$. Where $\propto$ is the angle of projection

48. A projectile having a horizontal range, R reaches a maximum height, H. prove that it must have been launched

i) with an initial speed equal to $\left[\frac{g(R^2+16H^2)}{8H}\right]^{½}$

ii) at an angle with the horizontal given by $\sin^{-1}\left[\frac{4H}{(R^2+16H^2)^{½}}\right]$

49. A ball is kicked from a point O so that it just clears two trees which are of height, h and at a distances x and y respectively from O. prove that if θ is the angle of projection of the ball,

i) $\tan\theta = \frac{h(x+y)}{xy}$

ii) the maximum height of the ball, $H = \frac{h(x+y)^2}{4xy}$

50. A particle is projected so as it just clears to walls each of height, h which lie at right angles with the plane of motion. The walls are at a distance, d apart and the first wall is at a distance, L from the point of projection. Show that the angle of elevation of the particle, ∝ is given by

$\tan\propto = \frac{h(2L+d)}{L(L+d)}$

51. A particle is projected from the top of the vertical cliff 160m high with a velocity of 180ms^{-1} at an angle of elevation of 30^{o}. Find the horizontal distance from the foot of the cliff to the point where it strikes the ground.

52. A particle is projected from a point height 3h above a horizontal plane, the direction of projection making an angle, ∝ with the horizon. Show that if the greatest height above the point of projection is h, the horizontal distance travelled before striking the plane is 6hcot ∝.

53. Two particles are projected simultaneously from the top and the bottom of a vertical cliff with angles of elevation ∝ and β respectively. They strike an object at the same point simultaneously. Show that if p is the horizontal distance of the object from the cliff, the height of the cliff is given by p(tanβ - tan∝)

54. Two boys stand on a horizontal ground at a distance, d apart. One throws a ball from a height, 2h with a velocity, V and the other catches it at a height, h above the horizontal at which the first boy throws the ball. Show that $gd^2 \tan^2\theta - 2V^2 d\tan\theta + gd^2 - 2V^2h = 0$ holds if $d = 2h\sqrt{2}$ and $V^2 = 2gh$. Hence calculate

i) the value θ

ii) the greatest attained by the ball in terms of h, u, g and θ

55. A bullet is fired out with the initial velocity of projection is $240ms^{-1}$ to the sea in a horizontal direction from a gun situated on the top of a cliff 78.4m high. Calculate

i) the distance at which the bullet will strike the water from the foot of the cliff.

ii) the inclination to the horizontal at which the bullet will strike the surface of the water.

56. a) A particle is projected at an angle of elevation of 30^o with a speed of $21ms^{-1}$, if the point of projection is 5m above the horizontal ground. Find the horizontal distance that the particle travels before striking the ground.

b) A boy throws a ball at an initial speed of $40ms^{-1}$ at an angle of elevation, $\propto$. Show taking $g = 10ms^{-2}$, that the times of flight corresponding to a horizontal range of 80m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$.

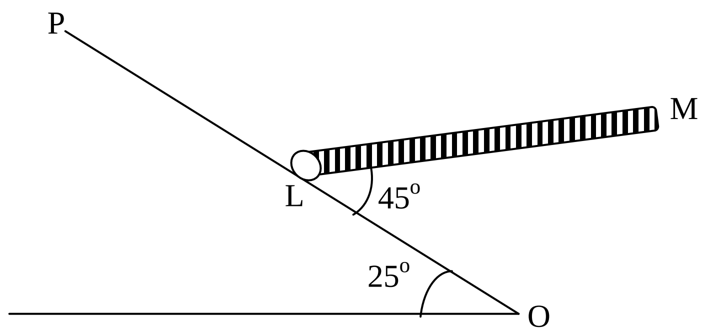
57. A particle is projected with a velocity of $40ms^{-1}$ at an angle of 60^o to the horizontal from the foot of the plane inclined at an angle of 30^o to the horizontal. Find the time at which the particle hits the plane.

58. A uniform ladder of length, 2L and weight, W rests in a vertical plane with one end against a rough vertical wall and the other end against a rough horizontal surface, the angles of friction at each end being $\tan^{-1}\frac{1}{3}$ and $\tan^{-1}\frac{1}{2}$ respectively.

i) if the ladder is in limiting equilibrium at either end. Find θ the angle inclination of the ladder to the horizontal.

ii) a man of weight 10 times that of the ladder begins to ascend it, how far will he climb before the ladder slips.

59. A uniform rod LM of weight, W rests with L on the smooth plane PO of inclination 25° as shown in the diagram below.



The angle between LM and the plane is 45°. What force parallel to PO applied at M will keep the rod in equilibrium? Give your answer in terms of W.

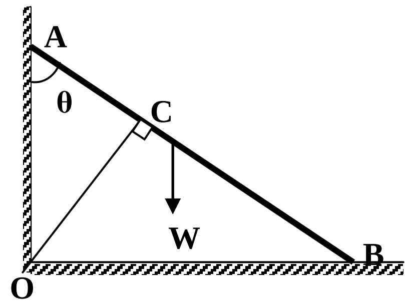
60. A non – uniform ladder AB, 10m long and mass 8kg lies in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end B resting against a smooth vertical wall. If the Centre of gravity of the ladder is 3m from the foot of the ladder and the ladder makes an angle of 30° with the horizontal. Find the

i) coefficient of friction between the ladder and the ground

ii) reaction at the wall

61. Two smooth rods AB and AC each of weight, W and length 10cm are smoothly hinged at A. the ends B and C rest on a smooth horizontal plane. An extensible string joins B and C and the system is kept in equilibrium in a vertical plane with the string taut. An object of weight, 2W climbs the rod AC to a point E such that AE = 8cm. given that angle BAC is 2θ. Determine in terms of W and θ the reaction at the ends B and C and the tension in the string. Hence show that the reaction at the hinge A is given by $\frac{W}{10}\sqrt{49\tan^{-1}\theta + 4}$

62. The diagram below shows a uniform rod AB of weight, W and length, L resting at an angle, θ against a smooth vertical wall at A. The other end B rests on a smooth horizontal table. The rod is prevented from slipping by an inelastic string OC, C being a point on AB such that OC is perpendicular to AB and O on the point of intersection of the wall and the table. Angle AOB is 90^o.

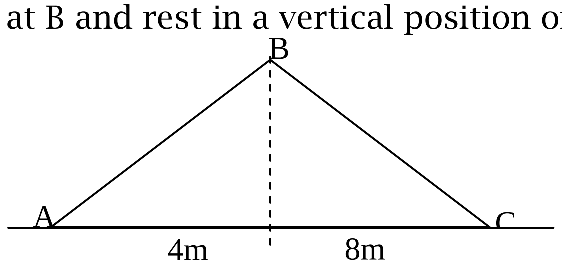


Find the

i) tension in the string

ii) reactions at A and B in terms of θ and W.

63. Two uniform rods AB and BC of masses 4kg and 6kg respectively are hinged at B and rest in a vertical position on the smooth floor as shown below.



A and C are connected by a rope

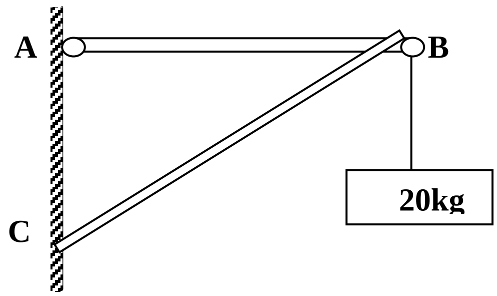
i) find the reactions between the rods and the floor at A and C when the rope is taut.

ii) if now a body is attached a quarter of the way up CB and the reactions are equal, find the mass of the body.

64. A non-uniform ladder AB is in equilibrium with A in contact with a horizontal floor and B in contact with a vertical wall. The ladder is in a vertical plane perpendicular to the wall. The Centre of gravity of the ladder is at G where AG = $\frac{2}{3}$AB. The coefficient of friction between the ladder and the wall is twice that between the ladder and the floor. If the

ladder makes an angle, θ with the wall and the angle of friction between the ladder and the floor is λ, prove that $4\tan\theta = 3\tan 2\lambda$. How far can a man of mass, m ascend the ladder without the ladder slipping given that $\theta = 45^o$ and the coefficient of friction between the ladder and the floor is ½.

65. The diagram below shows a uniform horizontal plank AB of length 3m and mass 2kg hinged to a vertical wall at A and supported at B by a light rod CB hinged to the same wall at C such that AC = 4m



If a mass of 20kg hangs from B, find the

i) tension force in the rod CB

ii) the magnitude of the reaction at A.

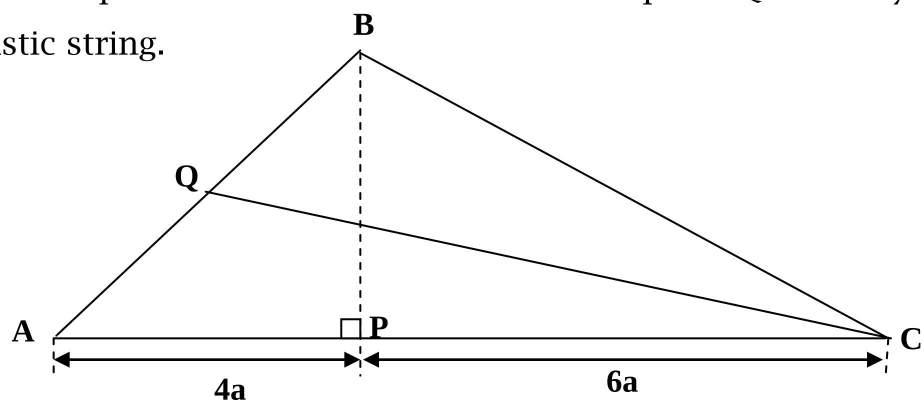
66. Two uniform rods AB and BC of equal length but of mass, M and 3M respectively are freely jointed together at B. the rods stand in a vertical plane with the ends A and C on a horizontal ground. The coefficient of friction, μ at the points of contact with the ground is the same and the rods are inclined at 60^o to each other. Given that one of the rods is on the point of sliding, find μ and the reaction at the hinge B when the rods are in this position.

67. A rod AB of length 0.6m long and mass 10kg is hinged at A. its centre of mass is 0.5m from A. a light inextensible string attached at B passes over a fixed smooth pulley 0.8m above A and supports a mass, M hanging freely. If a mass of 5kg is attached at B so as to keep the rod in a horizontal position, find the

i) value of M.

ii) reaction at the hinge.

68. The diagram below shows two uniform rods AB and BC of weights W and 3W respectively, which are smoothly hinged together at B. Point P is a point on AC which is vertically below B and AP = 4a, PC = 6a. The rods rest in equilibrium in vertical plane with the ends A and C on a smooth horizontal plane. The end C is connected to a point Q on AB by a light inelastic string.



Show that the magnitude of the reaction of the plane at A is $\frac{17}{16}$W and find the magnitude of the reaction of the plane at C. if BC = 10a and Q is the midpoint of AB, find the tension in CQ.

69. A non-uniform ladder AB of length 12m and mass 30kg has its Centre of gravity at the point of trisection of its length, nearer to A. the ladder rests with end A on the rough horizontal ground (coefficient of friction ¼) and B against a rough vertical wall (coefficient of friction $^1/_5$. The ladder makes an angle, θ with the horizontal such that tan θ = $\frac{9}{4}$. A straight horizontal string connects A to a point at the base of the wall vertically below B. what tension must the string be capable of withstanding if a man of mass 90kg is to reach the top of the ladder safely.

70. A non-uniform metallic beam AB of mass 30kg and length 4.4m balances with 45kg mass placed at end B. when support Q is placed 1.2m from B, find how far from end A the weight of the beam acts. If the beam balances again when an additional mass of 22.3kg is hang at end A and another support P is placed 0.8m from end A, determine the reactions at P and Q.

NAMILYANGO COLLEGE

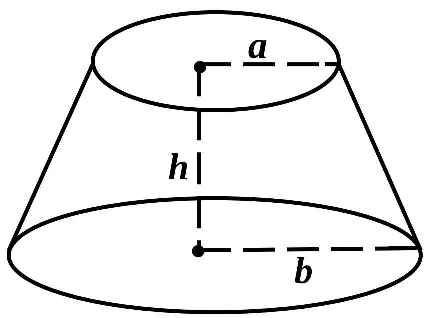
A –LEVEL APPLIED MATHEMATICS SEMINAR 2024

PAPER STRUCTURE	***SECTION A***	***SECION B***
Statistics and probability	***3***	***3***
Numerical methods	***2***	***2***
Static Mechanics	***1***	***1***
Dynamic Mechanics	***1***	***1***
Kinematic Mechanics	***1***	***1***

STATIC MECHANICS

1(a) *Show that the centre of gravity of a solid cone of radius* **r** *and height* **h** *lies along its axis at a distance* $\frac{1}{4}h$ *from the base*

(b) *The figure below shows a solid conical frustum of height* **h** *and whose top and bottom radii being* **a** *and* **b** *respectively*



Show that the centre of gravity of this frustum lies along its axis at a distance

$\dfrac{h(b^2+2ab+3a^2)}{4(b^2+ab+a^2)}$ *from the bottom*

2. (a) *A particle of weight* **50N** *is supported by two light inextensible strings of lengths* **8m** *and* **13m** *attached to two fixed points* **15m** *apart on a horizontal beam. Find the tension in each string*

(b) *The ends* **P** *and* **Q** *of a light inextensible string* **PBCQ** *are fastened to two fixed points on a horizontal beam.* *Particles of mass* **3kg** *and* **4kg** *are attached to the string at the points* **B** *and* **C** *respectively. If* **PB** *is inclined at* $45°$ *to the horizontal and* $\angle PBC = 150°$*, find the:*

(i) *tension in each portion of the string*

(ii) *angle* **CQ** *makes with the horizontal*

3. ***ABCDE*** *is a regular pentagon of side* **4m.** *Forces of magnitude* **2N, 3N, 5N** *and* **7N** *act along* $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ *and* $\overrightarrow{EB}$ *respectively. The resultant of this system of forces cuts* **AB** *produced at* **H.** *Taking* **A** *as the origin and* **AB** *as the* **x–axis,**

(i) *find the magnitude and direction of the resultant force*

(ii) *show that length* $AH = 15{\cdot}34m$ *correct to* **4 S·f**

(iii) *find the perpendicular distance from* **A** *to the line of action of the resultant force*

4. *Coplanar forces* $(3i + 3j)N$, $(4i - 5j)N$, $(-5i + 2j)N$ *and* $(2i + 3j)N$ *act at points with position vectors* $(3i + j)m$, $(i + 3j)m$, $(-2i + j)m$ *and* $(-2i - 2j)m$ *respectively.*

(i) *Find the resultant force and find where its line of action cuts the* **x–axis**

(ii) *A couple of moment* **bNm** *acting anticlockwise and a force* $(pi + qj)N$ *acting at a point with position vector* $(2i + j)m$ *are now added to the above system. If these reduce the system to equilibrium, find the values of* **p, q** *and* **b**

5. *A uniform ladder* **PQ** *of length* **2a** *and weight* **w** *is inclined at an angle of* $\tan^{-1}2$ *to the horizontal with its end* **Q** *resting against a smooth vertical wall and end* **P** *on a rough horizontal ground with which the coefficient of friction is* $\frac{5}{12}$. *If a boy of weight* **W** *can safely ascend a distance* **x** *up this ladder before it slips,*

(i) *show that* $x = \dfrac{a(2w + 5W)}{3W}$

(ii) *deduce that the boy can only reach the top of the ladder if* **W = 2w**

6. *A uniform rod* **PQ** *of length* **8m** *and weight* **18N** *is freely hinged at* **P** *and carries a mass of* **3kg** *at* **Q.** *The rod is kept horizontally by a string attached at* **Q** *and to a point* **C** *distant* **6m** *vertically above* **P.** *Find the:*

(i) *tension in the string*

(ii) *magnitude and direction of the reaction at the hinge*

7. *A box of mass* **6·5kg** *is placed on a rough plane inclined at* $\tan^{-1}\left(\frac{3}{4}\right)$ *to the horizontal. The coefficient of friction between the box and the plane is* **0·25.** *Find the least horizontal force required:*

(i) *to move the box up the plane*

(ii) *to prevent the box from sliding down the plane*

KINEMATIC MECHANICS

1. *Two cyclists* ***P*** *and* ***Q*** *are travelling along straight roads which cross at an angle of* $60°$ *at point* ***C.*** *If their riding speeds towards* ***C*** *are* $4kmh^{-1}$ *and* $5kmh^{-1}$ *and they are respectively* ***8km*** *and* ***15km*** *from* ***C,*** *find the:*

(i) *least distance between the cyclists*

(ii) *time that elapses before the cyclists are closest*

(iii) *distances of* ***P*** *and* ***Q*** *from* ***C*** *when they are nearest*

2. *A battleship and a patrol ship are initially* ***16km*** *apart with the battleship on the bearing of* $035°$ *from the patrol. The battleship sails at* $14kmh^{-1}$ *in the direction* ***S30°E*** *and the patrol ship at* $17kmh^{-1}$ *in the direction* ***N50°E.***

(a) *Find the:*

(i) *shortest distance between the ships*

(ii) *time that elapses before the ships are closest*

(b) *If the guns on the battleship have a range of up to* ***6km,*** *find the time that elapses when the patrol ship is within range of these guns*

3. *Two boats* ***P*** *and* ***Q*** *are sailing with respective speeds of* $20kmh^{-1}$ *and* $19kmh^{-1}$.

Initially ***P*** *is* ***10km*** *from* ***Q*** *on a bearing of* $320°$ *and is on a course of* $200°$.

Find the:

(i) *two possible courses* ***Q*** *can take in order to intercept* ***P***

(ii) *time taken for interception to occur in each case*

4. (a) *At certain times, the position vector* **r** *and velocity vector* **v** *of two ships* **A** *and* **B** *are as follows:*

$$r_A = (-2i + 3j)km \quad V_A = (12i - 4j)kmh^{-1} \text{ at } 11:45am$$
$$r_B = (8i + 7j)km \quad V_B = (2i - 14j)kmh^{-1} \text{ at } 12:00noon$$

If the ships maintain these velocities, find the:

(i) *position vector of ship* **A** *at noon*

(ii) *time when the ships are closest*

(iii) *shortest distance between the ships*

(iv) *distance of ship* **A** *from the origin when the two ships are closest*

(b) *If instead ship* **B** *had a velocity* $V_B = (-2i - 14j)kmh^{-1}$, *show that the ships will collide and find when and where the collision occurs*

5. *Two stations* **P** *and* **Q** *are* $2{\cdot}5km$ *apart. A train passes* **P** *at a speed of* $14ms^{-1}$ *and accelerates uniformly for* **20s** *to a speed* v_1. *Over the next* **720m** *covered in* **15s,** *its acceleration alters to a speed* v_2. *It travels at this speed for* **13s** *and then over the next* **500m** *covered in* **10s** *with uniform deceleration its speed at* **Q** *is* v_3. *Find the:*

(i) *values of* v_1, v_2 *and* v_3

(ii) *acceleration for the second part of the motion*

(iii) *fraction of the whole distance covered with constant speed*

6. ***(a)*** *The velocity of a uniformly accelerating train changes from* $\boldsymbol{u}$ *to* $\boldsymbol{v}$ *in time* $\boldsymbol{t}$.

(i) *Sketch its velocity time graph*

(ii) *Derive the equation for its motion* $\boldsymbol{v^2 = u^2 + 2as}$, *where* $\boldsymbol{a}$ *is its acceleration*

(b) *A uniformly accelerating train passes successive kilometer marks with velocities* $\boldsymbol{20ms^{-1}}$ *and* $\boldsymbol{28ms^{-1}}$ *respectively. Find its velocity when passing the next kilometer mark*

(c) *A uniformly retarding car takes* **8s** *and* **16s** *to travel between successive points* **A, B** *and* **C** *each* **144m** *apart. Find the further distance it travels to come to rest*

7. ***(a)*** *A particle is projected from level ground at an angle of elevation* $\boldsymbol{\theta}$ *with initial speed* $\boldsymbol{ums^{-1}}$. *Show that the equation of its path is given by*

$$y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$$

(b) *A ball kicked from level ground with a speed of* $\boldsymbol{30ms^{-1}}$ *just clears a vertical wall* **9m** *high and* **72m** *away. Calculate the possible angles of projection* (*use* $\boldsymbol{g = 10ms^{-2}}$)

(c) *A ball projected at an angle with a speed of* $\boldsymbol{14\sqrt{10}ms^{-1}}$ *from the top of a tower* **200m** *high hits the ground at a point* **200m** *away from the foot of the tower.*

(i) *Show that the two possible directions of projection are at right angles to each other*

(ii) *Find the two possible times of flight*

8. *A particle is projected from the origin* **O** *with velocity* $\boldsymbol{u} = (9\cdot 8\boldsymbol{i} + 29\cdot 4\boldsymbol{j})ms^{-1}$ *and moves freely under gravity.*

(a) *Find the particle's velocity and position vector after* **t** *seconds*

(b) *Show that the particle's equation of path is given by* $y = 3x - \frac{5x^2}{98}$.

Hence find the particle's horizontal range and maximum height reached

(c) *Find the direction in which the particle is moving after* **t** *seconds*

(d) *Find the two times when the direction in which the particle is moving is at right angles with the line joining the position of the particle to* **O**

DYNAMIC MECHANICS

1. *A force* $(24t\boldsymbol{i} - 12\boldsymbol{j})N$ *acts on a particle of mass* **2kg** *initially at rest at a point with position vector* $(-4\boldsymbol{i} + 3\boldsymbol{j})m$. *Find the:*

(i) *velocity of the particle after* **t** *seconds*

(ii) *distance from the origin after* **2s**

(iii) *power exerted by the force at* **t = 2s**

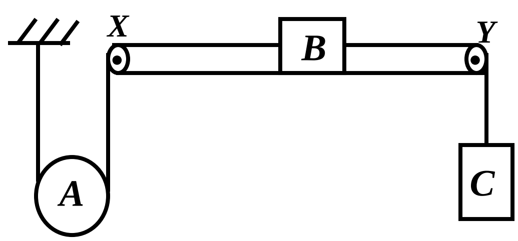
(iv) *work done by the force between* **t = 1s** *and* **t = 2s**

2. *A pile–driver of mass* $m_1 kg$ *falls through a height* **hm** *onto a pile of mass* $m_2 kg$ *without rebounding. If the pile is driven into the ground a depth* **d m,** *show that the resistance of the ground to penetration* $R = \frac{m_1^2 gh}{(m_1 + m_2)d} + (m_1 + m_2)g$ *and the time for which the pile is in motion* $T = \frac{(m_1 + m_2)d\sqrt{2gh}}{m_1 gh}$

3. (a) *A car of mass **500kg** is moving up a hill inclined at* $\sin^{-1}\left(\frac{1}{7}\right)$ *to the horizontal. The resistance to motion of the car is **300N.** If the power output of the car is **84kW,** find the acceleration of the car when its speed is* $35ms^{-1}$

(b) *A car of mass **800kg** is moving at a constant speed of* $20ms^{-1}$ *down a hill inclined at an angle* θ *to the horizontal. The resistance to motion of the car is **1300N.** If the power output of the car is **10kW,** show that* $\sin\theta = \frac{5}{49}$

4. *A light inelastic string is fixed at one end and passes under a moveable pulley **A** of mass **4kg** which hangs vertically. The other end of the string is attached to particle **B** of mass **4kg** which lies on a rough horizontal table. A second inelastic string connects **B** to a freely hanging particle **C** of mass **10kg.** The strings are passing over smooth fixed pulleys **X** and **Y** as shown*



*If the system is released from rest and the coefficient of friction between **Q** and the table is **0·5,** find the:*

(i) *accelerations of **A, B** and **C***

(ii) *tension in the strings*

(iii) *reaction of pulley **Y** on the string*

5. *A particle moving with S·H·M has velocities of* $7\cdot 5ms^{-1}$ *and* $4ms^{-1}$ *as it passes through points* **P** *and* **Q** *which are* **0·9m** *and* **0·2m** *respectively from the end points of its path. Find the:*

(i) *length of its path and the period of the motion*

(ii) *maximum velocity and maximum acceleration*

(iii) *time it takes to travel directly from* **P** *to* **Q**

(iv) *time which elapses before it next passes through* **Q**

(v) *mean velocity during its motion from one extreme position to the other*

6. *A particle of mass* **m** *is suspended from a fixed point* **O** *by a light elastic string of natural length* **l**. *When the particle hangs in equilibrium, the extension of the string is* **d.** *The particle is then slightly vertically displaced from its equilibrium position and then released. Show that it moves with SHM of period* $2\pi\sqrt{\dfrac{d}{g}}$

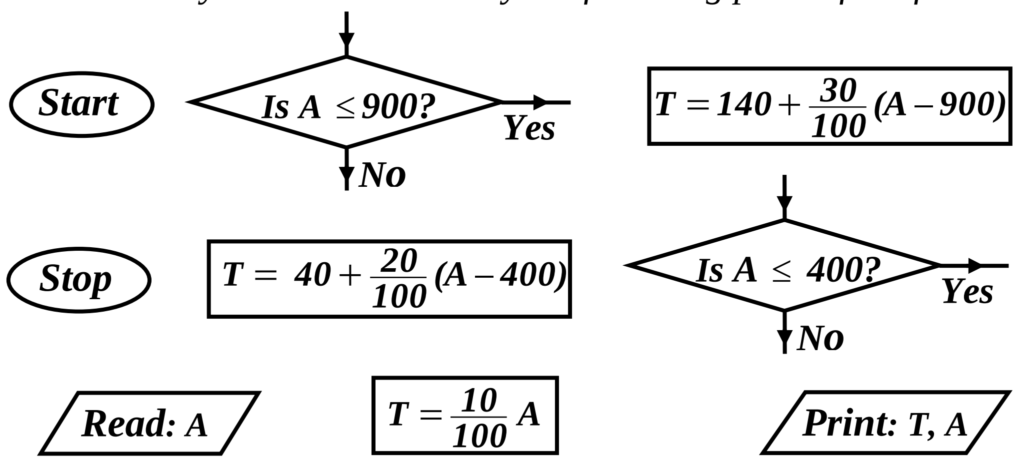
NUMERICAL METHODS

1. *Three decimal numbers* **X, Y** *and* **Z** *were rounded off to give* **x, y** *and* **z** *with errors* **Δx, Δy** *and* **Δz** *respectively. Show that the maximum relative error in the approximation of* $\dfrac{X}{Y-Z}$ *by* $\dfrac{x}{y-z}$ *is* $\left|\dfrac{\Delta x}{x}\right| + \left|\dfrac{\Delta y}{y-z}\right| + \left|\dfrac{\Delta z}{y-z}\right|$. *Hence find the absolute error and percentage error in* $\dfrac{1\cdot 6}{2\cdot 15 - 1\cdot 9}$ *and the interval within which its exact value is expected to lie*

2. *The income tax of an employee is calculated as follows:*

Taxable Income, A (£)	***Tax rate (%)***
01 – 400	***10***
401 – 900	***20***
Above 900	***30***

The taxation system is described by the following parts of the flowchart:



(i) *Arrange the given parts to form a complete logical flowchart*

(ii) *State the purpose of the flowchart*

(iii) *Performing a dry run for the flow chart and complete the table below*

A	***T***
1500	- - - -
750	- - - -

3. (i) *Show that the Newton Raphson formula for finding the natural logarithm of the* k^{th} *root of a number* A *is given by* $x_{n+1} = x_n - \frac{1}{k} + \frac{A}{k}e^{-kx_n}$

(ii) *Draw a flowchart that computes and prints the root in* ***(a)*** *above correct to* ***3*** *decimal places*

(iii) *Perform a dry run for your flowchart using* $x_o = 1 \cdot 25$, $A = 147$ *and* $k = 4$

4. Find the percentage error in estimating $\int_{0}^{\frac{\pi}{3}} \sec^2 x\, dx$ using trapezium rule with six ordinates correct to **4** decimal places and state how it can be reduced

5. An equation has two iterative formulae $x_{n+1} = 2x_n^2 e^{x_n}$ and $x_{n+1} = \frac{1}{2}e^{-x_n}$.

(i) Use each formula twice to deduce with a reason the most suitable one when $x_o = 0 \cdot 4$. Hence state the root correct to **3** d.p

(ii) Without iterating deduce with a reason the most suitable formula when $x_o = 0 \cdot 4$. Hence use it twice to find the root correct to **3** d.p.

(iii) Show that the equation for the two iterative formulae is $2xe^{x} - 1 = 0$

6.(a) Show that the equation $\cos(x^2) - x + 3 = 0$ has a root between **2·5** and **3.** Hence use linear interpolation thrice to find the root correct to **3** d.p

(b) In a motor rally, car **P** was observed to be at distances of **350m** and **400m** from the starting line when the chasing car **Q** was at distances of **240m** and **300m** respectively. How far was car **P** from the starting line when car **Q:**

(i) started chasing it

(ii) caught up with it

(c) Use the fact that $f(1{\cdot}15) = 1{\cdot}32$ and $f^{-1}(1 \cdot 26) = 1 \cdot 25$ to find the value of $f^{-1}(1 \cdot 22)$ by linear interpolation correct to **3 d.p**

7. *Locate the ranges where the two real roots of the equation* $x^4 - x - 10 = 0$ *lie. Hence use Newton Raphson method to find the least root correct to* **3** *d·p*

STATISTICS

1. *The table below shows the prices of three items for the years* ***2023*** *and* ***2024***

Item	PRICE (£)		Weights
	IN 2023	IN 2024	
A	150	153	5
B	250	261	2
C	525	588	3

Taking ***2023*** *as the base year, calculate the:*

(i) *simple aggregate price index for* ***2024.*** *Comment on your result*

(ii) *weighted mean price index for* ***2024.*** *Comment on your result*

(iii) *weighted aggregate price index for* ***2024.*** *Comment on your result*

(iv) *cost of items in* ***2023,*** *similar to the items in* ***2024*** *whose cost was* ***£540*** *using the result in* ***(iii)*** *above*

2. *The grades of* ***8*** *students in* ***UNEB, pre mock*** *and* ***post mock*** *were as follows:*

UNEB	B	A	O	C	B	E	O	D
Pre Mock	D	B	D	C	A	F	O	E
Post Mock	C	B	E	O	A	D	F	E

(a) *Calculate the rank correlation coefficient between the grades of:*

(i) UNEB *and* ***Pre Mock***

(ii) UNEB *and* ***Post Mock***

(b) *Which of the two mocks had a better correlation with* ***UNEB?*** *Give a reason*

3. *The table below shows the weights in kg of* **100** *babies***:**

Weights	***2***	***2·5***	***4·5***	***6***	***7***	***8***
No of babies	***35***	***20***	***20***	***10***	***10***	***5***

(i) *Calculate the mean, variance and median for the above data*

(ii) *Assuming this was a sample taken from a normal population***,** *find the* **90%** *confidence interval for the mean weight of all babies*

4. *The table below shows the weights in kg of* **40** *boys***:**

Weights	***30 –< 35***	***35 –< 40***	***40 –< 55***	***55 –< 60***	***60 –< 65***
Frequency	***8***	***5***	***12***	***9***	***6***

(a) *Calculate the mean, mode and percentage of boys heavier than* **45 kg**

(b) *Draw an ogive for the data and use it to estimate the***:**

(i) *median weight*

(ii) *quartile deviation*

(iii) *range of the weights of the middle* **70%** *of the boys*

(c) *Draw a histogram for the data and use it to estimate the modal weight*

5. *The price index of an item in* **2022** *based on* **2023** *was* **88.** *Its price index in* **2024** *based on* **2023** *was* **132.** *Find its***:**

(i) *price index in* **2024** *based on* **2022**

(ii) *price in* **2022** *if its price in* **2024** *was* **£600**

PROBABILITY

1. *A box contains* **150** *red and* **50** *blue pens.* **48** *pens are drawn in succession at random from the box with replacement. Find the probability of picking:*

(i) *exactly* **30** *red pens*

(ii) *at least* **29** *red pens*

(iii) *at least* **8** *but less than* **20** *blue pens*

2. *A continuous* **r·v** $X \sim R(3, 15)$.

(a) *Write down the p·d·f of* **X** *and sketch it*

(b) *Find:* **(i)** $E(X)$ **(ii)** $Var(X)$ **(iii)** *the upper quartile of* **X**

(iv) $P(4 < X < 10)$ **(v)** $P(|X - 7| < 2 / X \geq 6)$

(vi) *the distribution function of* **X** *and sketch it*

3. *Box* **A** *contains* **4** *red and* **3** *green pens, box* **B** *contains* **3** *red and* **4** *green pens, while box* **C** *contains* **5** *red and* **2** *green pens. Boxes* **A, B** *and* **C** *are in the ratio* **2 : 3 : 5** *respectively as likely to be picked. If a box is selected at random and two pens are picked from it without replacement,*

(a) *find the probability of picking:*

(i) *pens of different colours*

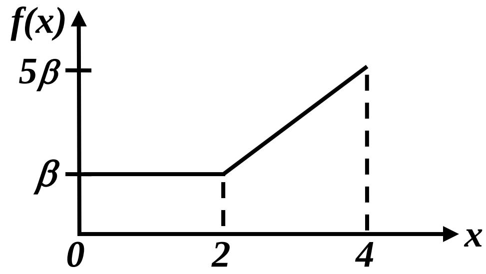
(ii) *box* **B** *given that the pens drawn are of the same colour*

(b) *If* **X** *is the number of red pens drawn, find the:*

(i) *probability distribution of* **X**

(ii) *median, mean and variance of* **X**

4. *The p·d·f of a continuous* **r·v X** *is distributed as follows:*



Find:

(i) *the value of* $\boldsymbol{\beta}$

(ii) *the equations of the p·d·f*

(iii) *the mean and median of* **X**

(iv) *the cumulative distribution function of* **X** *and sketch it*

(v) $P(X > 1 / X < 3)$

5. *A continuous r·v* **X** *has the following distribution function:*

$$F(x) = \begin{cases} 0 & , \quad x \le 0 \\ \dfrac{\sqrt{2}}{2}\sin x & , \quad 0 \le x \le \lambda \\ 1 - \dfrac{\sqrt{2}}{2}\cos x & , \quad \lambda \le x \le \dfrac{\pi}{2} \\ 1 & , \quad x \ge \dfrac{\pi}{2} \end{cases}$$

(a) *Show that* $\lambda = \dfrac{\pi}{4}$

(b) *Find:*

(i) $P\left(\left|X - \dfrac{\pi}{4}\right| \le \dfrac{\pi}{12}\right)$

(ii) *the equations of the p·d·f and sketch it, hence deduce the mean of* **X**

(iii) $E\left(3X - \dfrac{\pi}{3}\right)$

6. *A discrete* **r·v X** *has the following p·d·f***:**

$$P(X = x) = \begin{cases} \frac{1}{60}(ax+b) & , \quad x = 1,2,3,4 \\ 0 & , \quad otherwise \end{cases}$$

Given that $F(3) = \frac{13}{20}$**,** *find the***:**

(i) *values of* **a** *and* **b,** *hence sketch the p·d·f of* **X**

(ii) $P(X > 1 / X < 3)$

(iii) *mean and variance of* **X**

(iv) $E(3X - 4)$ *and Var*$(3X - 4)$

7. (a) *A random variable* **X** *is binomially distributed with mean* **4·8** *and variance* **2·88.** *Find* $P(X < 6)$

(b) *A student answers* **12** *questions***.** *The chance of passing each question is* $\frac{1}{3}$**.**

*Find the probability of passing***:**

(i) *exactly* **7** *questions*

(ii) *at least* **2** *questions*

8. *A random variable* **X** *is normally distributed such that* $P(X < 76) = 0{\cdot}9772$ *and* $P(72 < X < 76) = 0{\cdot}044$**.** *Find***:**

(i) *the mean and standard deviation of* **X**

(ii) $P(X > 45)$

(iii) *the interval which contains the middle* **95%** *of distribution*

9. *A random sample of* ***100*** *nails taken from a normal population had the following lengths* $\boldsymbol{x}$ *in cm:* $\sum x = 380$ *and* $\sum x^2 = 1840$. *Find the:*

(i) *unbiased estimate for the population variance*

(ii) 90·8% *confidence interval for the population mean*

10. *A random sample of* ***36*** *items drawn from a normal population is such that the* ***95%*** *confidence interval for the mean of all the items is* ***[67·9, 77·7].*** *Find the* ***90%*** *confidence limits for the mean of all the items*

11. *Given that* $P(A \cup B) = \frac{9}{10}$, $P(A/B) = \frac{1}{3}$ *and* $P(B/A) = \frac{2}{5}$, *find:*

(i) P(A)

(ii) $P(\overline{A}/\overline{B})$

(iii) P(A *or* ***B*** *but not both* ***A*** *and* ***B)***

12. *At a certain party,* ***25%*** *of the guests are women. Nile beer and Bell beer are the only drinks available for the guests.* ***40%*** *of the women and* ***70%*** *of the men take Nile beer. Of the men taking Nile beer,* ***80%*** *got drunk and of the men taking Bell beer,* ***60%*** *got drunk. Of the women taking Nile beer,* ***50%*** *got drunk and of the women taking Bell beer,* ***40%*** *got drunk. Find the probability that a randomly selected guest:*

(i) *takes Nile beer*

(ii) *got drunk*

(iii) *got drunk given that is a woman*

(iv) *is a man given that he got drunk for taking Nile beer*

13. ***(a)*** *Events* **A** *and* **B** *are such that* $P(A)=\frac{2}{3}$, $P(B)=\frac{1}{4}$ *and* $P(A\cup B)=\frac{17}{24}$.

Find: ***(i)*** $P(A\cap B)$ ***(ii)*** $P(\overline{A}\cap B)$ ***(iii)*** $P(\overline{A}\cap\overline{B})$ ***(iv)*** $P(\overline{A}\cup\overline{B})$

(b) *If* $\overline{A}$ *and* $\overline{B}$ *are independent events,*

(i) *Show that the events* **A** *and* **B** *are also independent*

(ii) *find* $P(B)$ *and* $P(\overline{A}\cup\overline{B})$, *if* $P(A)=0\cdot375$ *and* $P(A\cup B)=0\cdot75$

(c) *Find the possible values of* $P(A)$, *if* **A** *and* **B** *are independent events such that* $P(A\cap B)=0\cdot3$ *and* $P(A\cup B)=0\cdot875$

14. *A task in mathematics is given to three students whose chances of solving it are* $\frac{1}{3}$, $\frac{1}{4}$ *and* $\frac{1}{5}$ *respectively. Find the probability that:* ***(i)*** *the task is solved* ***(ii)*** *only one student solves it* ***(iii)*** *at least two of them solved it*

15. *Two soldiers* **A** *and* **B** *in that order take turns shooting a bullet at a target. The first one to hit the target wins the game. If their chances of hitting the target on each occasion they shoot are* $\frac{1}{3}$ *and* $\frac{1}{4}$ *respectively, find the chance that:*

(i) **A** *wins the game on his third shot* ***(ii)*** **A** *wins the game.*

16. *Mutually exclusive events* **A** *and* **B** *are such that* $P(A\cup B)=0\cdot75$ *and* $P(A)=0\cdot27$, *find:* ***(i)*** $P(\overline{A}\cup B)$ ***(ii)*** $P(\overline{A}\cap\overline{B})$ ***(iii)*** $P(A\cap B)'$

17. *Exhaustive events* **A** *and* **B** *are such that* $5P(A)=4P(B)$ *and* $P(A\cap B)=\frac{1}{5}$.

Find: ***(i)*** $P(A)$ ***(ii)*** $P(A/B)$ ***END***

STATISTICS AND PROBABILITY:

1a) The probability that Amos goes to school on cloudy day is $\frac{7}{10}$ and $\frac{1}{5}$ on a clear day. If the probability of a cloudy day is $\frac{3}{5}$, find the probability that the day was cloudy given that he did not go to school.

b) If P and Q are independent event;

i) Show that P' and Q are also independent.

ii) Find $P(Q)$ given that $P(P)=0.25$ and $P(P\cup Q)=0.75$.

2. During the national teachers' conference, 40% of the delegates supported the idea of the 10% salary increase for the financial year 2019. Given that journalists randomly interviewed 450 delegates, find the probability that;

a) Less than 150 delegates supported the salary increase,

b) Between 160 and 170 delegates inclusive supported the salary increase.

3. The table below shows the heights measured in cm for a group of senior six students;

Height	177-186	187-191	192-196	197-201	202-206	207-216
Frequency	12	8	8	9	7	6

a) Draw a histogram. Hence state the modal class.

b) Calculate the (i) mean (ii) standard deviation. (ii) Mode

4. Events A, b and C are such that $P(A)=x$, $P(B)=y$ and $P(C)=x+y$. If $P(A\cup N)=0.6$ and $P\left(B/A\right)=0.2$,

(i) Show that $4x+5y=3$.

(ii) Given that B and C are mutually exclusive and that $P(B\cup C)=0.9$, determine another equation in x and y.

(iii) Hence find the values of x and y.

5. Of the 30 drivers interviewed, 9 have been involved in a car accident at some time. Of those who have been involved in the accident, 5 wear glasses. The probability of wearing glasses, given that the driver has not had a car accident is $\frac{1}{3}$. Find the probability that a;

(a) Person chosen at random wears glasses.

(b) Glasses wearer has been an accident victim.

6a) Given that A and B are two events such that $P(A')=0.3$, $P(B)=0.1$ *and* $P\left(A/B\right)=0.2$. Find (i) $P(A\cup B)$ (ii) $P\left(A/B'\right)$

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b) A box A contains 1 red, 3 green and 1 blue ball. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?

7. The cumulative distribution function of a discrete random variable x is as shown in the table below.

x	1	2	3	4
$F(x)$	0.14	0.47	0.79	1.00

Find the (i) $P(2 < x \le 4)$ (ii) median of x (iii) $P\left(x < 3 / 2 \le x < 4\right)$ (iv) mean of x.

8. The probability density function of a continuous random variable x is given by;

$$f(x) = \begin{cases} \frac{2}{13}(x+1), & 0 \le x \le a \\ \frac{2}{13}(5-x), & a \le x \le b \\ 0 & elsewhere \end{cases},$$ find the values of a and b. Hence, sketch $f(x)$.

(i) Calculate the mean and standard deviation of x.

(iii) Find $P(x < 2.5)$.

9. The Maths and Physics examination marks of a certain school are given in the following table.

Maths (x)	28	34	36	42	52	54	60
Physics(y)	54	62	68	70	76	68	74

a)i) Plot the marks on the scatter diagram and comment on the relationship between the two subjects.

ii) Draw a line of best fit and use it to predict the Physics mark of a student whose Maths mark is 50.

b) Calculate the rank correlation coefficient between the marks. Comment on the significance of Maths on Physics performance based on 5% level of significance

10a) A random sample of ten packets is taken. These have masses (Measured in kg) of x_1, x_2, ……, x_{10} such that; $\sum_{i=1}^{10} xi = 2.57$ and $\sum_{i=1}^{10} x^2 i = 0.6610$. Calculate a 95% confidence limits for the mean.

b) The weights of ball bearings are normally disturbed with mean 25gram and standard demotion 4 grams. If a random sample of 16 ball bearings is taken, find the probability that the mean of the sample his between 24.12 grams and 26.73 grams.

11. The table below the number of reported accidents to workers in a certain factory over the past 12 years.

a) Find the mean and median of the workers

Age of Worker	15-	20-	25-	30-	35-	40- < 45
Number of employees	42	52	28	20	18	16

b) Plot a cumulative frequency curve and use it to estimate the;

(i) Proportion of employees who are not more than 27 years old.
(ii) 80% central limits of the age of employees.

12. A survey of the mass (kg) of girls in the certain school in the final year was taken and the results are shown below;

Mass (kg)	< 45	<50	<60	<75	<80	<85	<90	<100	<105	<110
Cumulative frequency	0	9	25	46	63	77	82	89	93	94

(a) Display the information on a histogram and hence find the modal mass

(b) Calculate the standard deviation of the mass.

13. A random variable X has the probability density function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & elsewhere \end{cases}$$

a) Show that the variance of x is $\dfrac{(b-a)^2}{12}$.

b) If the expected value and variance of the distribution is 1 and $\dfrac{3}{4}$ respectively,

(i) $P(x<0)$

(ii) Value of a such that $P(X > a + \sigma) = \dfrac{1}{4}$. Where σ is the standard deviation of X.

14. A pharmacist had the following records of unit price and quantities of drugs sold for the years 2010 and 2011.

Drug	Unit price per carton		Quantities per carton	
	2010	**2011**	**2010**	**2011**
Aspirin	80	125	40	45
Panadol	100	90	70	90
Quinine	55	75	8	10
Coartem	90	100	10	10

Taking 2010 as the base year,

(i) Calculate the price relatives for each drug in 2011.
(ii) Obtain the simple aggregate price index number for 2011.

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(iii) Calculate the weighted price index and comment on it.
(iv) Calculate the weighted aggregate price index and comment on it.

MECHANICS:

15. A body of mass 0.2kg is acted upon by a force $\mathbf{F} = 8t\,\mathbf{i} - 4t^2\mathbf{j} + 2(3 - t^2)\mathbf{k}\ N$. Initially, the body is at rest at a point with position vector $-10\,\mathbf{i} + 12\mathbf{j} - 4\,\mathbf{k}$. Find the
(i) velocity after time t seconds,
(ii) Distance covered by the body in 2seconds.

16. An object of mass 4kg moves under the action of three forces $\mathbf{F}_1 = \begin{pmatrix} \frac{t}{2} - 1 \\ t - 3 \end{pmatrix} N$,

$\mathbf{F}_2 = \begin{pmatrix} \frac{t}{2} + 2 \\ \frac{t}{2} - 4 \end{pmatrix} N$ and $\mathbf{F}_3 = \begin{pmatrix} t - 3 \\ 3t - 2 \end{pmatrix} N$ at time t seconds. Find the
(i) Work done by the resultant force when the object is displaced from point $A(4,\ 5)$ to a point $B\,(7,\ -9)$ within a time of $1\,s$.
(ii) Size of the acceleration of the object after a time of $2\ s$.
(iii) Power developed when the object moves with velocity of $3\mathbf{i} - 6\mathbf{j}\ m\,s^{-1}$ after a time of $2\ s$.

17. Two smooth inclined planes meet at right angles, the inclination of one to the horizontal being 30° and the other being 60°. Bodies of masses 2kg and 4kg lie on the respective planes, and the two masses are joined by a light inextensible string passing over a smooth fixed pulley at the intersection of the planes.
(i) If the masses are released from rest, find the acceleration of the masses and tension of the string.
(ii) If the surface with 4kg mass experiences a frictional force of 0.25N, find the new acceleration of the masses when the system is set from rest.

18. Four forces represented by $i - 4j, 3i + 6j, -9i + j$ *and* $5i - 3j$, at a points with position vectors $3i - j, 2i + 2j, -i - j$ *and* $-3i + 4j$ respectively.

(a) Show that the forces reduce to a couple and find its moment
(b) If the fourth force is removed, find the magnitude and direction of the resultant forces.

19a) If the horizontal range of a particle projected with a velocity v is a, show that the greatest height x, attained is given by the equation $16gx^2 - 8v^2x + ga^2 = 0$, where g is acceleration due to gravity.

b) A small ball is projected from a point A on a level ground and passes at a height of 2metres over a point on the ground at a distance of 12m from A. The ball is caught at a height of 1metre by a girl who is 24metres from A. Find the angle of projection of the ball as it leaves point A.(g=10m/s^2)

20a) A particle of mass 2kg is pushed up a rough inclined plane whose slope is 3 in 5 by a horizontal force of 46.25N. The particle starts from rest at P and is moved up a line of greatest slope of the plane to the point Q, where PQ=15m. The coefficient of friction between the plane and the particle is 0.25. Find the speed acquired by the particle at Q (Take $g=10ms^{-2}$)

b) A pump draws water from a tank and delivers to a height of 10m at the end of a horse. The cross-sectional area of the horse is $10cm^2$ and the water leaves the end of the horse with a speed of $7.5\,m\,s^{-1}$. Find the rate at which the pump is working.

21a) A particle is projected at an angle of 30^0 with a speed of $21\,m\,s^{-1}$. If the point of projection is 5m above the horizontal grounds, find the horizontal distance that the particle travels before striking the ground.

b) A boy throws a ball at an initial speed of 40 ms^{-1} at an angle of elevation α. Show that the times of flight corresponding to the horizontal range of 8 m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$. (Take g=10 ms^{-2}).

22. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:

i) Acceleration of the system.

ii) Tension in the string.

(iii) Distance moved by the moveable pulley in 1.5 s.

23a) A body of mass 5 kg slides a distance of 8 m down a rough plane inclined at an angle of $\sin^{-1}\left(\frac{4}{5}\right)$to the horizontal. If the coefficient of friction is 0.4, find the velocity attained by the body.

b) A particle of mass 50 kg is suspended by two light inelastic strings of lengths 9 m and 12m attached to two points distant 15 m apart. Calculate the tensions in the strings.

24. A rod AB 1m long has a weight of 20N acting at a point 60cm from A. It rests horizontally with A hinged on a vertical wall. A string BC is fastened to the wall at C, 75cm vertically about A find the;

(i) Tension in the string

(ii) Reaction from at point A.

25a) A particle travelling in a straight line with the constant acceleration covers distances x and y in the 3rd and 4th seconds of its motion respectively. Show that its initial speed is given by U = $\frac{1}{2}(7x - 5y)$.

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b) Two stations A and B are a distance of $6x m$apart along a straight path. A train starts from rest at A and accelerates uniformly until it attains a speed of Vms^{-1} covering a distance of x m. The train then maintains this speed until it has travelled a further $3x$ m . It then retards uniformly to rest at B. show that if T is the time taken for a train to travel from A to B then T = $(\frac{9x}{V})$ s

26. Forces of magnitudes **6N, 6N, 4N, 10N** and **8N** act along **AB, BC, CD, DB** and **AD** respectively, in the directions indicated by the order of the letters of a rectangle **ABCD** of dimensions 4m by 3m. Find the;

(i) Magnitude of their resultant
(ii) Equation of the line of action of their resultant force
(iii) Distance from **B** where it cuts **AB.**

27a) A,B,C and D are the points (0,0),(10,0),(7,4) and (3,4) respectively. If AB, BC, CD and DA are made of a thin wire of uniform mass. Find the coordinates of the centre of gravity.

b)i) If instead ABCD is a uniform lamina, find its centre of gravity G

ii) If the lamina hung from B, find the angle AB makes with the vertical

28. An object of mass 5kg is initially at rest at a point whose position vector is -2i +j. If it is acted upon by a force, F = 2i +3j -4k, find

a) The acceleration

b) The velocity after 3s

c) Its distance from the origin after 3s.

29a) A car is being driven at $20\ m\ s^{-1}$on a bearing of 040°. the wind is blowing from 330° with a speed of $10\ m\ s^{-1}$. Find the velocity of the wind as experienced by the driver of the car.

b) At 8$a.m$ two boats A and B are 5.2km apart with A due west of B, and B travelling in a direction N20°w at a steady speed of 13kmh^{-1}. If A travels due north at 12kmh^{-1}, determine;

(i) The path of B relative to A

(ii) The distance between A and B at 8:30am.

30. A particle of mass M kg is attached to a string of natural length L and modulus of elasticity 4Mg. The other end is attached to a fixed point O. If the particle is released from rest at O.

(i) Find the greatest distance below O reached by the particle and its acceleration at this point.

(ii) Show that the speed of the particle is $1.5\sqrt{gL}$ when the acceleration is instantaneously zero.

NUMERICAL METHODS:

31a)The table below shows x and f(x);

x	50.24	48.11	46.93	44.06
f(x)	4.116	7.621	9.043	11.163

Use linear interpolation or extrapolation to estimate;
(i) x when f(x)=8.614
(ii) f^{-1} (5 1.07)

b) Use trapezium rule with six ordinates to find the value of $\int_1^2 \frac{x^2}{x^2+1}\,dx$, correct to three decimal places.

c) Find the exact value of $\int_1^2 \frac{x^2}{x^2+1}\,dx$, correct to three decimal places. How can you improve on the degree of accuracy?

32a) Use the trapezium rule with 6 strips to evaluate $\int_1^2 (x-1)\,In\;x\;dx$ correct to 4 decimal places.

b) Find the exact value of $\int_1^2 (x-1)\,In\;x\;dx$, Hence find the maximum possible error in your calculations in (a) above.

33. If the numbers x and y are approximations with errors Δx and Δy respectively. Show that the maximum absolute error in the approximations of x^2y is given by

$|\boldsymbol{2xy\Delta x}| + |\boldsymbol{x^2\Delta y}|$ Hence find the limits within which the true value of x^2y lies given that $x = 2.8 \pm 0.016$ and $y = 1.44 \pm 0.008$

34a) Show that the equation $3x^2 - x - 5 = 0$ has a real root between x= -1.4 and x= -1.2.
b) Use linear interpolation to estimate the initial root
c) Using the root in (b), use Newton – Raphson Method to find the root, correct to two decimal places.

35a) The dimensions of a rectangle are 8cm and 4.26.
(i) State the maximum possible error in each dimension.
(ii) Find the range within which the area of the rectangle lies. (correct to 2 decimal places)
b) The radius r and height h of a cylinder are measured with corresponding errors Δr and Δh respectively. Show that the maximum possible error in the volume is $\left|\frac{\Delta h}{\text{h}}\right| + 2\left|\frac{\Delta r}{\text{r}}\right|$

36a) Derive the simplest formula based on Newton Raphson's method to show that for the equation $3x = 1n3$ it satisfies $x_{r+1} = \frac{1}{3}\left\{\frac{e^{3x_r}(3x_r - 1) + 3}{e^{3x_r}}\right\}$

b) Draw a flow chart that;

i) reads the initial approximation x_0,

ii) Accepts estimations to 4 significant figures

iii) Prints the root to $4sf$.

iv) Perform a dry-run for your flow chart for $x_0 = \frac{1}{3}$.

37a) Show that the iterative formula based on Newton Raphson's Method for approximating the root of the equation $2\ln x = x - 1$ is given by;

$$x_{n+1} = x_n \left(\frac{2\ln x_n - 1}{x_n - 2}\right) \text{ n = 0, 1, 2. . . . .}$$

(b) Draw a flow chart that,

(i) Reads the initial approximation x_o of the root.

(ii) Computes and prints the root correct to two decimal places, using the formula in (a) above.

END

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STATISTICS AND PROBABILITY

1. (a) Let C and G be the event "it's a cloudy day" and "Amos goes to school" respectively. $P(C)=\frac{3}{5}$, $P(^{G}/_{C})=\frac{7}{10}$, $P\left(^{G}/_{C^1}\right)=\frac{1}{5}$

$$P\left(^{C}/_{G^1}\right)=\frac{P(CnG^1)}{P(G^1)}=\frac{P(C)xP(^{G^1}/_{C})}{P(CnG^1)+P(C^1nG^1)}=\frac{\frac{3}{5}x\frac{3}{10}}{\frac{3}{5}x\frac{3}{10}+\frac{2}{5}x\frac{4}{5}}=\frac{9}{25}$$

(b) (i) For independent events $P(PnQ)=P(P)xP(Q)$

$$P(P)=P(PnQ)+P(PnQ^1)$$

$$(PnQ^1)=P(P)-P(P)xP(Q)=P(P)[1-P(Q)]=P(P)xP(Q^1)$$

Hence P and Q^1 are also independent events

(ii) $P(PuQ)=P(P)+P(Q)-P(PnQ)$

$$0.75=0.25+P(Q)-0.25xP(Q) \quad P(Q)=\frac{0.5}{0.75}=\frac{2}{3}\ or\ 0.6667$$

2. $n=40, p=\frac{40}{100}=\frac{2}{5}, q=\frac{3}{5}$

Let x be a r.v "number of delegates supporting the salary increase

$x{\sim}B\left(450,\frac{2}{5}\right)$ Since n>20 then, $x{\sim}N(\mu,\sigma^2)$

But $\mu=450x\frac{2}{5}=180,\ \sigma^2=450x\frac{2}{5}x\frac{3}{5}=108$

(a) $P(x<150)=P\left(z<\frac{149.5-180}{\sqrt{108}}\right)$

$$=P(z<-2.935)=0.5-\emptyset(2.935)=0.5-0.0017=0.4983$$

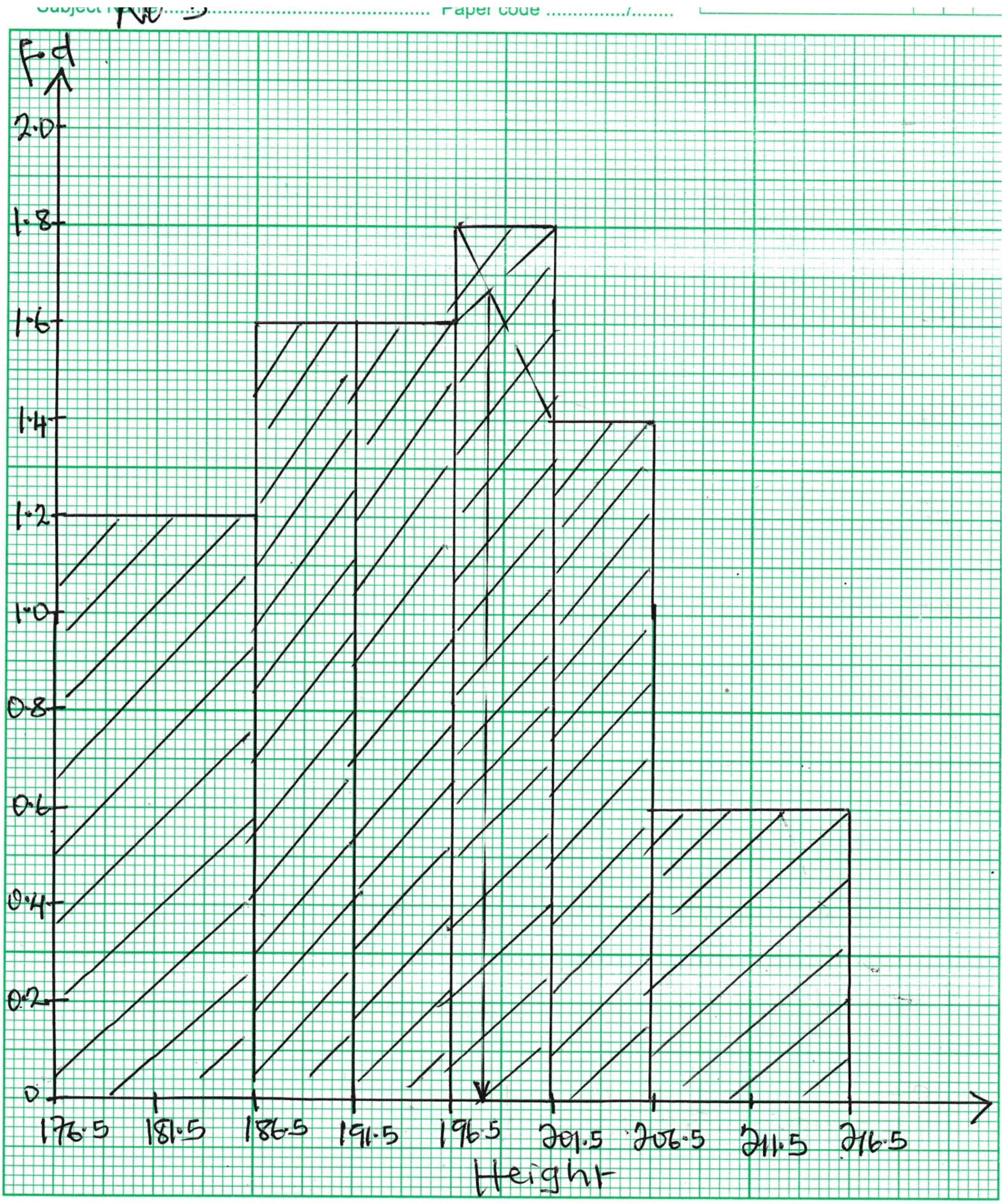
(b) $P(160<x<170)=P\left(\frac{159.5-180}{\sqrt{108}}<z<\frac{170.5-180}{\sqrt{108}}\right)$

$$=P(-1.973<z<-0.914)=\emptyset(0.914)-\emptyset(1.973)=0.1804-0.0243$$
$$=0.1561$$

3.

Class boundaries	Mid-point, x	f	fx	fx²	Freq. density
176.5 - 186.5	181.5	12	2178	395307	1.2
186.5 – 191.5	189	8	1512	285768	1.6
191.5 – 196.5	194	8	1552	301088	1.6
196.5 – 201.5	199	9	1791	356409	1.8
201.5 – 206.5	204	7	1428	291312	1.4
206.5 – 216.5	211.5	6	1269	268393.5	0.6
		$\sum f=50$	$\sum fx=9730$	$\sum fx^2=1898277.5$	

(a) Modal class is (197 – 201)



(a) (i) Mean, $\bar{x} = \frac{9730}{50} = 194.6cm$

(ii) Standard deviation $= \sqrt{\frac{1898277.5}{50} - (194.6)^2} = 9.8178cm$

(iii) Mode$=196.5 + \frac{0.2}{0.2+0.4}x5 = 198.1667cm$

4. (i) $P(B/_A) = \frac{P(BnA)}{P(A)} = 0.2$

   But $PAuB) = P(A) + P(B) - P(AnB)$

   $0.6 = x + y - P(AnB) \quad \therefore P(AnB) = (x+y) - 0.6$

   Hence $\frac{(x+y)-0.6}{x} = 0.2 \quad \therefore 4x + 5y = 3 \ldots\ldots\ldots (i)$

   (ii) $P(BuC) = P(B) + P(C) \quad \rightarrow 0.9 = y + (x+y) \quad \therefore 10x + 20y = 9 \ldots\ldots\ldots (ii)$

   (iii) Solving (i) and (ii) simultaneously gives $x = \frac{1}{2} and\ y = \frac{1}{5}$

5. Let A and G be events the person has been involved in an accident and wears glasses respectively. $P(A) = \frac{9}{30}, P(G/_A) = \frac{5}{9}$ and $P\left(G/_{A^1}\right) = \frac{1}{3}$

   (a) $P(G) = P(AnG) + P(A^1nG) \quad \rightarrow P(G) = \frac{9}{30}x\frac{5}{9} + \frac{21}{30}x\frac{1}{3} = \frac{8}{15}$

   (b) $P(A/_G) = \frac{P(AnG)}{P(G)} = \frac{\frac{9}{30}x\frac{5}{9}}{\frac{8}{15}} = \frac{5}{16}$

6. (a) (i) $P(A/_B) = \frac{P(AnB)}{P(B)} \quad 0.2 = \frac{P(AnB)}{0.1} \quad \therefore P(AnB) = 0.02$

   Hence $P(AuB) = P(A) + P(B) - P(AnB) = 0.7 + 0.1 - 0.02 = 0.78$

   (ii) $P\left(A/_{B^1}\right) = \frac{P(AnB^1)}{P(B^1)} = \frac{P(A)-P(AnB)}{P(B^1)} = \frac{0.7-0.02}{0.9} = 0.7556$

   (b) Let A, B and G be events for picking “box A”, “box B” and a “green ball” respectively. $P(A) = \frac{1}{6}, P(B) = \frac{5}{6}, P(G/_A) = \frac{3}{5} and\ P(G/_B) = \frac{1}{5}$

   $P(A/_G) = \frac{P(AnG)}{P(G)} = \frac{P(AnG)}{P(AnG)+P(BnG)} = \frac{\frac{1}{6}x\frac{3}{5}}{\frac{1}{6}x\frac{3}{5}+\frac{5}{6}x\frac{1}{5}} = \frac{3}{8}$

7.

x	1	2	3	4
F(x)	0.14	0.47	0.79	1.00
P(X=x)	0.14	0.33	0.32	0.21

(i) $P(2 < x \leq 4) = F(4) - F(2) = 1 - 0.47 = 0.53$

or $P(2 < x \leq 4) = P(X=3) + P(X=4) = 0.32 + 0.21 = 0.53$

(ii) Median, $F(x) \geq 0.5$ Hence median = 3

(iii) $P\left(x < 3/_{2 \leq x < 4}\right) = \frac{P(x<3\ n\ 2\leq x<4)}{P(2\leq x<4)} = \frac{P(x=2)}{P(x=2)+P(x=3)} = \frac{0.33}{0.33+0.32} = 0.5077$

(iv) $Mean = 1x0.14 + 2x0.33 + 3x0.32 + 4x0.21 = 2.6$

8. (i) $At\ x = a,\quad \frac{2}{13}(a+1) = \frac{2}{13}(5-a) \quad \therefore a = 2$

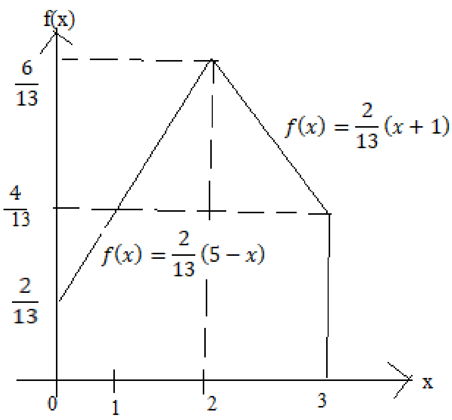
$$\int_0^2 \frac{2}{13}(x+1)dx + \int_2^b \frac{2}{13}(5-x)dx = 1$$
$$\frac{2}{13}\left[\left(\frac{x^2}{2}+x\right)_0^2 + \left(5x-\frac{x^2}{2}\right)_2^b\right] = 1$$

$$\frac{2}{13}\left[(2+2) + \left(5b-\frac{b^2}{2}\right) - (10-2)\right] = 1$$
$$b^2 - 10b + 21 = 0 \ \therefore b = 7 \ or\ b = 3$$

Check: $b = 7, \int_2^7 \frac{2}{13}(5-x)dx = \frac{2}{13}\left[5x-\frac{x^2}{2}\right]_2^7 = \frac{2}{13}\left[\left(35-\frac{49}{2}\right)-(10-2)\right] = \frac{5}{13}$

$b = 3, \int_2^3 \frac{2}{13}(5-x)dx = \frac{2}{13}\left[5x-\frac{x^2}{2}\right]_2^3 = \frac{2}{13}\left[\left(15-\frac{9}{2}\right)-(10-2)\right] = \frac{5}{13}$

Hence $a = 2, b = 3\ or\ 7$



$0 \le x \le 2$

$f(0) = \frac{2}{13}(0+1) = \frac{2}{13}, \qquad f(2) = \frac{2}{13}(2+1) = \frac{6}{13}$

$2 \le x \le 3$

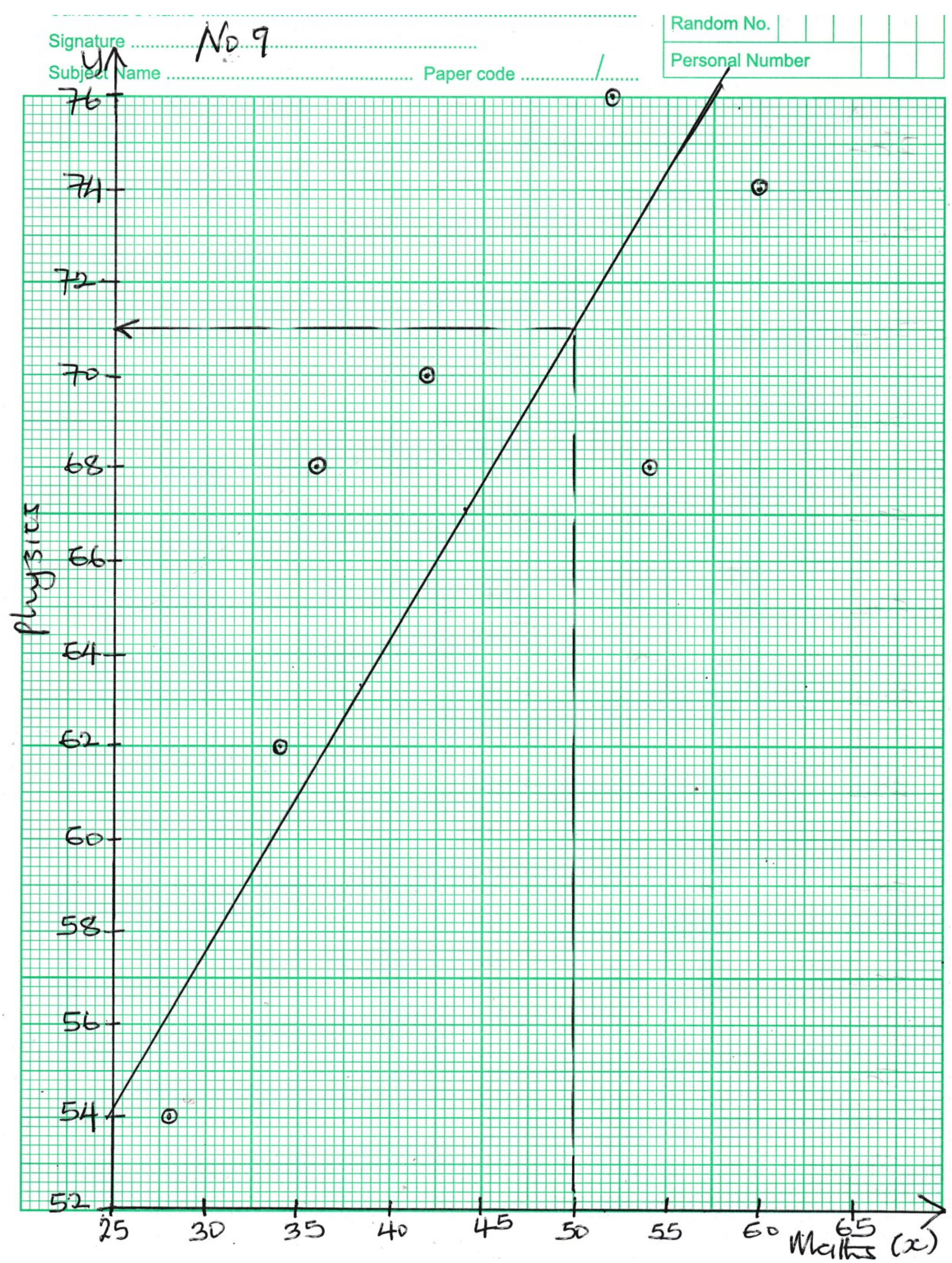
$f(2) = \frac{2}{13}(5-2) = \frac{6}{13}, \qquad f(3) = \frac{2}{13}(5-3) = \frac{4}{13}$

(ii) $E(x) = \int_0^2 \frac{2}{13}x(x+1)dx + \int_2^3 \frac{2}{13}x\,(5-x)dx$

$$= \frac{2}{13}\left[\left(\frac{x^3}{3}+\frac{x^2}{2}\right)_0^2 + \left(\frac{5x^2}{2}-\frac{x^3}{3}\right)_2^3\right]$$
$$= \frac{2}{13}\left[\left(\frac{2^3}{3}+\frac{2^2}{2}\right) + \left(\frac{5(3)^2}{2}-\frac{3^3}{3}\right) - \left(\frac{5(2)^2}{2}-\frac{(2)^3}{3}\right)\right] = \frac{5}{3} = 1\frac{2}{3}$$

(iii) $P(x < 2.5) = \int_0^2 \frac{2}{13}(x+1)dx + \int_2^{2.5} \frac{2}{13}(5-x)dx$

$$= \frac{2}{13}\left[\left(\frac{x^2}{2}+x\right)_0^2 + \left(5x-\frac{x^2}{2}\right)_2^{2.5}\right]$$
$$= \frac{2}{13}\left[(2+2) + \left(5x2.5-\frac{2.5^2}{2}\right) - (10-2)\right] = 0.8269$$



9. (a) (i) There is a positive relation between Physics and Maths
(ii) When Maths = 50, Physics = 71

Maths(x)	Physics(y)	R_x	R_y	D^2
28	54	7	7	0
34	62	6	6	0
36	68	5	4.5	0.25
42	70	4	3	1
52	76	3	1	4
54	68	2	4.5	6.25
60	74	1	2	1
				$\sum D^2 = 12.5$

$$\rho = 1 - \frac{6x12.5}{7(7^2-1)} = 0.7768$$

Since |ρ|=0.7768>0.75(table value), then there is significance of Maths on Physics performance at 5% level of significance based on 7 pairs of observations

10. (a) Sample mean $\bar{x} = \frac{2.57}{10} = 0.257$ Population mean $\hat{\mu} = \bar{x} = 0.257$

Sample variance $s^2 = \frac{0.6610}{10} - (0.257)^2 = 0.000051$

Population Variance $\hat{\sigma}^2 = \frac{10}{9}x0.000051 = 0.00057$

$95\%\ Confidence\ limits = 0.257 \pm 1.96x\frac{0.000057}{\sqrt{10}} = (0.256996,\ 0.257003)$

(b) Let x be a r.v " weight of ball bearings $x \sim N(25, 4^2)$

$$P(24.12 < \bar{x} < 26.73) = P\left(\frac{24.12 - 25}{\frac{4}{\sqrt{16}}} < z < \frac{26.73 - 25}{\frac{4}{\sqrt{16}}}\right)$$

$$= P(-0.88 < z < 1.73)$$

$$= \emptyset(1.73) + \emptyset(0.88)$$

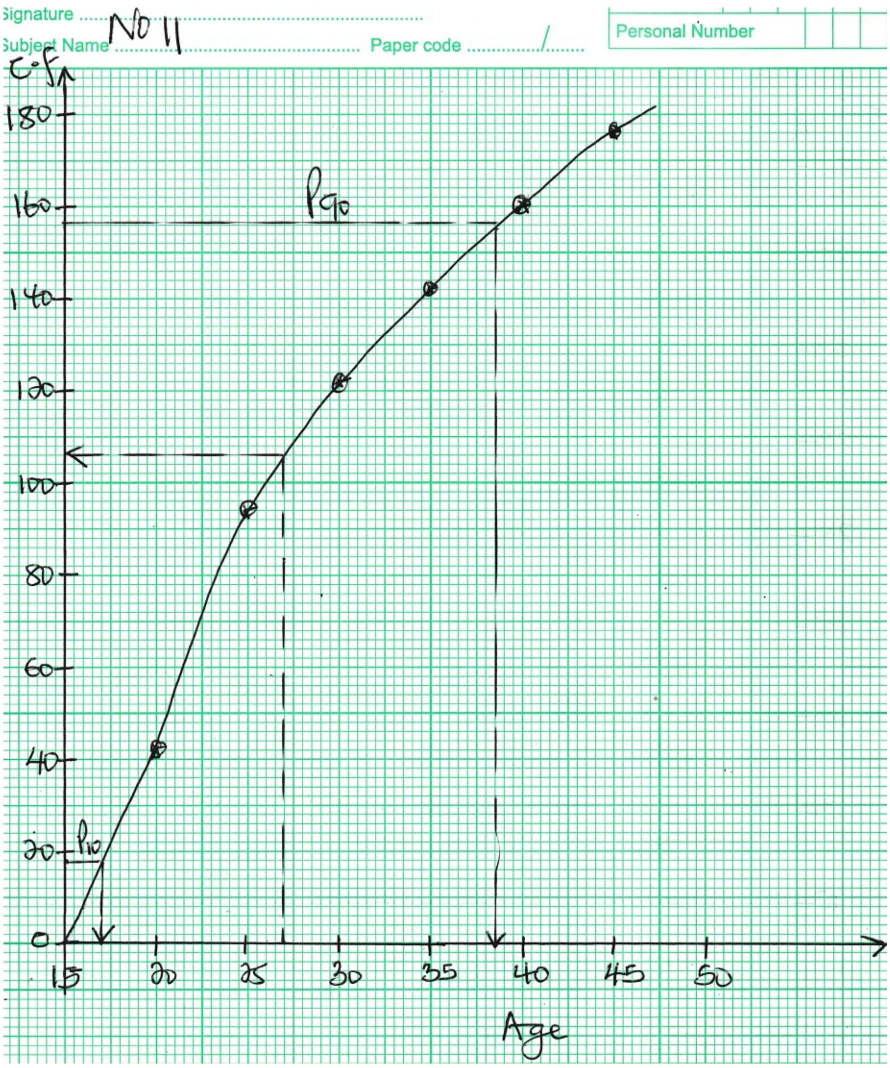
$$= 0.4582 + 0.3106$$

$$= 0.7688$$

11.

Class boundaries	Mid-point, x	f	fx	c.f
15 – 20	17.5	42	735	42
20 - 25	22.5	52	1170	94
25 - 30	27.5	28	770	122
30 – 35	32.5	20	650	142
35 – 40	37.5	18	675	160
40 - 45	42.5	16	680	176
		$\sum f = 176$	$\sum fx = 4680$	

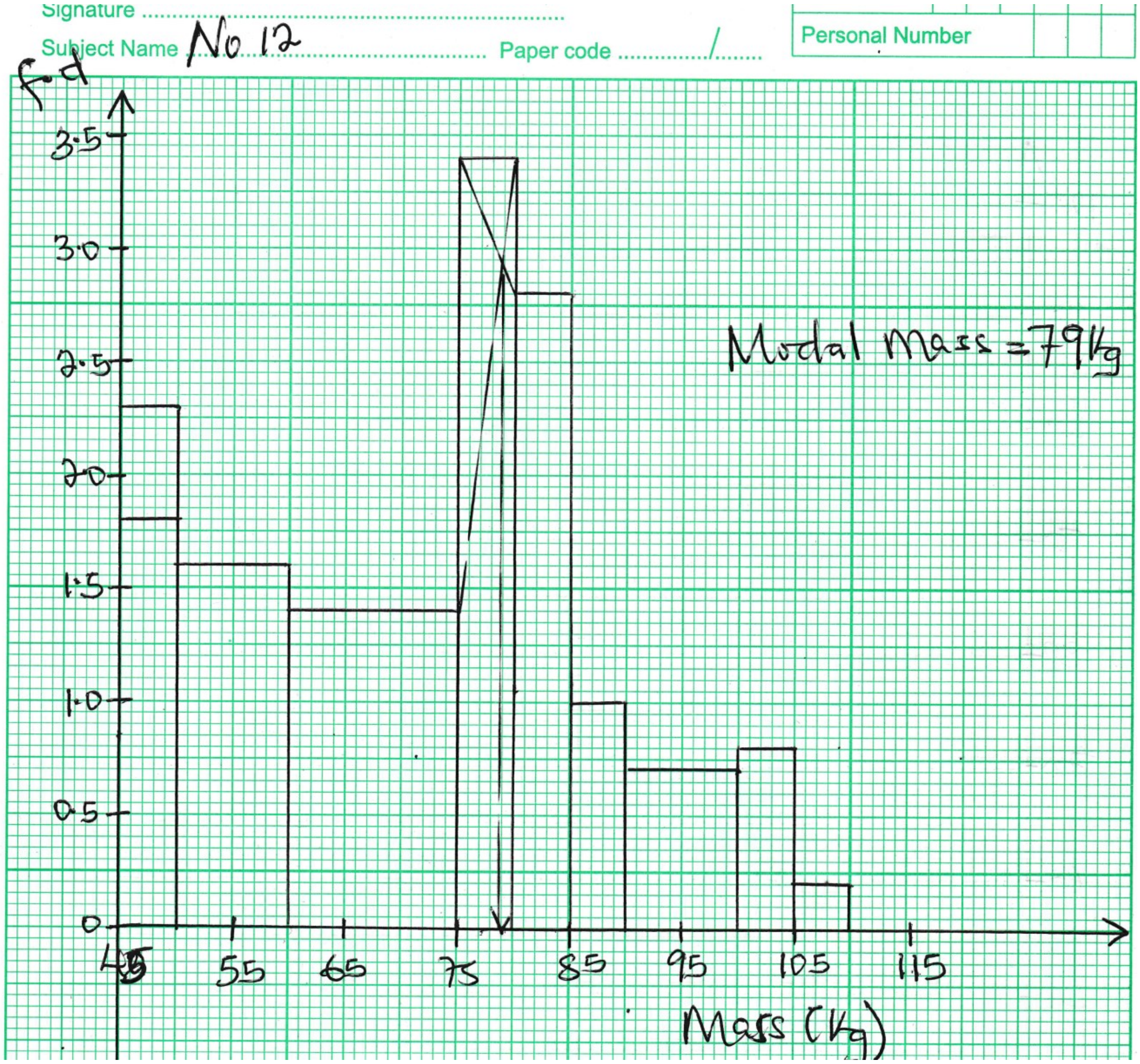
(a) Mean $=\frac{4680}{176} = 26.5909$ Median $=25 + \frac{\left(\frac{176}{2}-94\right)}{28}x5 = 23.9286$



(b) (i) Proportion more than 27 years = $\frac{106}{176} = \frac{53}{88}$

(ii) 80% Central limits = $[38.5, 17.0]$

12.



Class boundaries	Mid-point, x	f	fx	fx^2	Freq. density
45 - 50	47.5	9	427.5	20306.25	1.8
50 - 60	55	16	880	48400	1.6
60 - 75	67.5	21	1417.5	95681.25	1.4
75 - 80	77.5	17	1317.5	102106.25	3.4
80 - 85	82.5	14	1155	95287.5	2.8
85 - 90	87.5	5	437.5	38281.25	1.0
90 - <100	95	7	665	63175	0.7
100 - <105	102.5	4	410	42025	0.8
105 - <110	107.5	1	107.5	11556.25	0.2
		$\sum f = 94$	$\sum fx = 6817.5$	$\sum fx^2 = 516818.75$	

$$Standard\ deviation\ = \frac{516818.75}{94} - \left(\frac{6817.5}{94}\right)^2 = 15.4261$$

13. Mean, E(x) = $\int_a^b \frac{1}{b-a} x dx$

$$= \left[\frac{x^2}{2(b-a)}\right]_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{(a+b)}{2}$$

E(x^2) $= \int_a^b \frac{1}{(b-a)} x^2$ dx

$$= \left[\frac{x^3}{3(b-a)}\right]_a^b = \frac{b^3 - a^3}{3(b-a)} = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} = \frac{b^2+ab+b^2}{3}$$

Hence Variance = E(x^2) – $(E(x))^2$

$$= \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^2+ab+a^2}{3} - \frac{b^2+2ab+a^2}{4}$$

$$= \frac{4b^2+4ab+4a^2-3b^2-6ab-3a^2}{12} = \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}$$

b) Mean: $1 = \frac{b+a}{2}$

$b + a = 2$………………. (i)

Variance: $\frac{3}{4} = \frac{(b-a)^2}{12}$

$(b-a)^2 = 9$

$b - a = 3$……………………(ii)

Solving (i) and (ii) simultaneously gives;

b = 2.5 and **a = -0.5**

(i) P(X<0) $= \int_{-0.5}^{0} \frac{1}{(2.5--0.5)} dx = \frac{1}{3}[X]_{-0.5}^{0} = \frac{(0--0.5)}{3} = \mathbf{\frac{1}{6}}$

(ii) $P(x > a + \sigma) = \frac{1}{4}$ But $\sigma = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{4}$

$\int_{a+\frac{\sqrt{3}}{4}}^{2.5} \frac{1}{3} dx = \frac{1}{4} \rightarrow \left[\frac{x}{3}\right]_{a+\frac{\sqrt{3}}{4}}^{2.5} = \frac{1}{4} \rightarrow \frac{2.5-(a+\frac{\sqrt{3}}{4})}{3} = \frac{1}{4} \rightarrow a + \frac{\sqrt{3}}{4} = 2.5 - \frac{3}{4}$

$\therefore a = 0.8840$

14. (i) **Price relatives**= $\left(\frac{P_{2011}}{P_{2010}} x100\right)$

Aspirin: $\frac{125}{80} x100 = \mathbf{156.25}$, Panadol: $\frac{90}{100} x100 = \mathbf{90}$,

Quinine: $\frac{75}{55} x100 = \mathbf{136.3636}$, Coartem: $\frac{100}{90} x100 = \mathbf{111.1111}$

(ii) Simple aggregate price index $= \frac{\sum P_{2011}}{\sum P_{2010}} x100 = \frac{(125+90+75+100)}{(80+100+55+90)} x100$

$= \frac{390}{325} x100$

$= 120$

(iii) Weighted price Index $= \frac{\sum\left(\frac{P_{2011}}{P_{2010}}\right)w}{\sum W} x100 = \frac{\left(\frac{125}{80}x45+\frac{90}{100}x90+\frac{75}{55}x10+\frac{100}{90}x10\right)}{45+90+10+10} x100$

$= 113.5871$

Hence the prices in 2011 increased by 13.5871%

(iv) Weighted aggregate Price Index $= \frac{\sum(P_{2011})w}{\sum(P_{2010})w} x100$

$= \frac{(125x45+90x90+75x10+100x10)}{(80x40+100x70+55x8+90x10)} x100$

$= \frac{15475}{11540} x100 = 134.0988$

Hence the price in 2011 increased by 34.0988%

MECHANICS

15. (a) $F = ma$ $\quad 0.2a = 8t\tilde{\boldsymbol{i}} - 4t^2\tilde{\boldsymbol{j}} + 2(3 - t^2)\tilde{\boldsymbol{k}}$

$$a = (40t\tilde{\boldsymbol{i}} - 20t^2\tilde{\boldsymbol{j}} + 10(3 - t^2\boldsymbol{k})$$

Hence $v = \int adt \;\; \rightarrow v = \int (40t\tilde{\boldsymbol{i}} - 20t^2\tilde{\boldsymbol{j}} + 10(3 - t^2\boldsymbol{k})dt$

$$v = 20t^2\boldsymbol{i} - \frac{20}{3}t^3\boldsymbol{j} + 10(3t - \frac{t^3}{3})\boldsymbol{k} + c$$

When t=0s, c= 0. $\therefore\ v = 20t^2\boldsymbol{i} - \frac{20}{3}t^3\boldsymbol{j} + 10(3t - \frac{t^3}{3})\boldsymbol{k}$

(b) $r(t) = r(o) + v.t$

$$r(t) = (-10\,i + 12j - 4k) + \left[20t^2\boldsymbol{i} - \frac{20}{3}t^3\boldsymbol{j} + 10\left(3t - \frac{t^3}{3}\right)\boldsymbol{k}\right]\boldsymbol{t}$$

$$r(t) = (20t^3 - 10)i + \left(12 - \frac{20}{3}t^4\right)j + \left(30t^2 - \frac{10}{3}t^4 - 4\right)k$$

When $t = 2s$, $\quad r(2) = (20x8 - 10)i + \left(12 - \frac{20}{3}x16\right)j + \left(30x4 - \frac{10}{3}x2^4 - 4\right)k$

$$r(2) = 150i - \frac{284}{3}j + \frac{188}{3}k$$

$\therefore Distance, |r(2)| = \sqrt{(150^2 + (-\frac{284}{3})^2 + \left(\frac{188}{3}\right)^2}$ = 188.1194m

16. (i) $F = \begin{pmatrix} \frac{t}{2} - 1 \\ t - 3 \end{pmatrix} + \begin{pmatrix} \frac{t}{2} + 2 \\ \frac{t}{2} - 4 \end{pmatrix} + \begin{pmatrix} t - 3 \\ 3t - 2 \end{pmatrix} = \begin{pmatrix} 2t - 2 \\ \frac{9t}{2} - 9 \end{pmatrix} N$

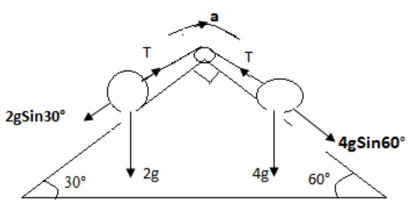
Work done = F.$\overline{AB}$ = $\begin{pmatrix} 2x1 - 2 \\ \frac{9x1}{2} - 9 \end{pmatrix} . \left[\begin{pmatrix} 7 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ -\frac{9}{2} \end{pmatrix} . \begin{pmatrix} 3 \\ -14 \end{pmatrix} = 0 + 63 = 63J$

(ii) F=ma, $4a = \begin{pmatrix} 2x2 - 2 \\ \frac{9x2}{2} - 9 \end{pmatrix} \quad 4a = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad a = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$

Hence $|a| = \sqrt{(0.5)^2 + 0}\ \therefore |a| = 0.5m/s^2$

(iii) Power=F.V $\ Power = \begin{pmatrix} 2x2 - 2 \\ \frac{9x2}{2} - 9 \end{pmatrix} . \begin{pmatrix} 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} . \begin{pmatrix} 3 \\ -6 \end{pmatrix} = 6 + 0 = 6W$

17. (a)



4kg mass: 4gsin60°-T = 4a ……….(1)

2kg mass: T – 2gSin30°=2a ………(2)

Adding (1) and (2)

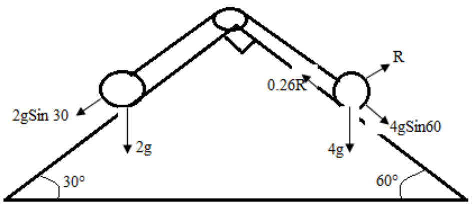
4gsin60° - 2gSin30° =6a

a= 4.0247 m/s²

Tension: T = 2(4.0247 +9.8Sin30°)

= **17.8494N**

(b)



4kg mass: 4gsin60°- 0.25R-T = 4a, But R = 4gCos60°

4gsin60° - 0.25x4gCos60°-T = 4a…….(3)

2kg mass: T – 2gSin30°=2a ………(4)

Adding (3) and (4)

4gsin60° - 0.25x4gCos60°- 2gSin30° =6a

a= 3.2080 m/s²

18. (a) Resultant force, $F = \begin{pmatrix}1\\-4\end{pmatrix} + \begin{pmatrix}3\\6\end{pmatrix} + \begin{pmatrix}-9\\1\end{pmatrix} + \begin{pmatrix}5\\-3\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

Moment, M $= \begin{vmatrix}3 & -1\\1 & -4\end{vmatrix} + \begin{vmatrix}2 & 2\\3 & 6\end{vmatrix} + \begin{vmatrix}-1 & -1\\-9 & 1\end{vmatrix} + \begin{vmatrix}-3 & 4\\5 & -3\end{vmatrix}$

$= (-12+1) + (12-6) + (-1-9) + (9-20)$

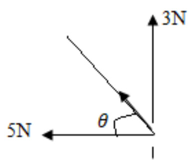
= -26Nm.

Since the resultant, F = 0 and the moment, M=26Nm in Clockwise direction, then the forces reduce to a couple

(b) The fourth force, $F_4 = \begin{pmatrix}5\\-3\end{pmatrix} N$, hence the resultant force of the remaining forces is

$F = \begin{pmatrix}-5\\3\end{pmatrix}$

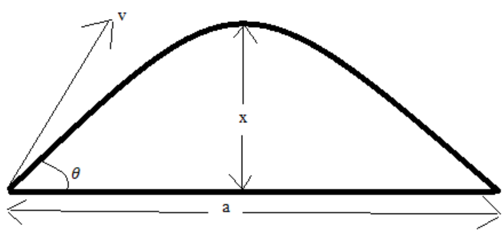
Magnitude, $|F| = \sqrt{(-5)^2 + 3^2} = \sqrt{34}$ = 5.8310N



$\tan\theta = \frac{3}{5}$ $\therefore \theta = 30.96°$

Hence the resultant force is 5.8310N acting at angle of 300.96° or N29.04°W

19. (a)



Range, $a = (v\cos\theta).\frac{2v\sin\theta}{g} = \frac{2v^2\sin\theta\cos\theta}{g} = \frac{v^2\sin 2\theta}{g}$

$\therefore \sin 2\theta = \frac{ag}{v^2} \quad \rightarrow \cos\theta = \frac{\sqrt{v^4-(ag)^2}}{v^2}$

Maximum height, $x = \frac{v^2\sin^2\theta}{2g}$ But $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$x = \frac{v^2.\frac{1}{2}(1 - \frac{\sqrt{v^4-(ag)^2}}{v^2})}{2g}$$

$$4gx = v^2 - \sqrt{v^4-(ag)^2})$$

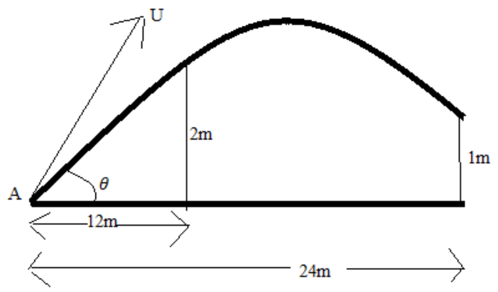
$$(4gx - v^2)^2 = (\sqrt{v^4-(ag)^2})^2$$

$$16g^2x^2 - 8gv^2x + v^4 = v^4 - a^2g^2$$

$$16g^2x^2 - 8gv^2x + a^2g^2 = 0$$

$$16gx^2 - 8v^2x + a^2g = 0$$

(b)



From equation of a trajectory; $y = xtan\theta - \frac{gx^2(1+tan^2\theta)}{2u^2}$

$$\textbf{Case 1}: \quad 2 = 12tan\theta - \frac{10x12^2(1+tan^2\theta)}{2U^2}$$

$$1 = 6tan\theta - \frac{360(1+tan^2\theta)}{U^2}$$

$$(1+tan^2\theta) = \frac{(6tan\theta - 1)U^2}{360} \ldots\ldots\ldots\ldots\ldots..(i)$$

$$\textbf{Case 2}: 1 = 24tan\theta - \frac{10x24^2(1+tan^2\theta)}{2U^2}$$

$$1 = 24tan\theta - \frac{2880(1+tan^2\theta)}{U^2} \quad \rightarrow (1+tan^2\theta) = \frac{(24tan\theta-1)U^2}{2880} \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

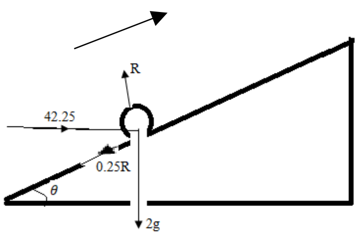
Equating (i) and (ii)

$$\frac{(6tan\theta - 1)U^2}{360} = \frac{(24tan\theta - 1)U^2}{2880}$$

$$8(6tan\theta - 1) = (24tan\theta - 1)$$

$$48tan\theta - 24tan\theta - 7 = 0 \quad \rightarrow tan\theta = \frac{7}{24} \quad \therefore \theta = 16.3°$$

20. (a)



Using F =ma,

Along the inclined plane:

$42.25Cos\theta - 0.25R - 2gSin\theta = 2a$, But R = 2g$Cos\theta$

$42.25Cos\theta - 0.25x2gCos\theta - 2gSin\theta = 2a$, $Sin\theta = \frac{3}{5}$ hence $Cos\theta = \frac{4}{5}$

$$a = \frac{42.25x\frac{4}{5} - 2x10(0.25x\frac{4}{5} + \frac{3}{5})}{2} = 8.9m/s^2$$

PQ = 15m = s, u =0m/s

From $v^2 = u^2 + 2as$, Hence $v^2 = 0 + 2x8.9x15 \quad \therefore v = 16.3401m/s$

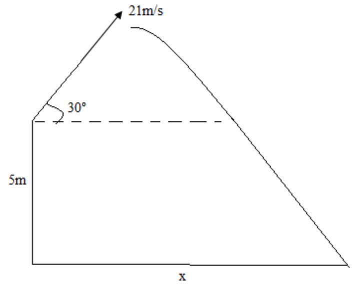
(b) Volume per second, $v = speedxarea = 7.5x\frac{10}{10{,}000} = 7.5x10^{-3}m^3/s$

Mass per second, $m = densityxvolume\ per\ second = 1000x7.5x10^{-3} = 7.5kg/s$

Work done per second = $K.E + P.E = \frac{1}{2}x7.5x7.5^2 + 7.5x9.8x10 = 945.9375J/s$

Hence Power done = 945.9375W

21. (a)



Vertical displacement: $-5 = (21sin30°)t - \frac{1}{2}x9.8xt^2$

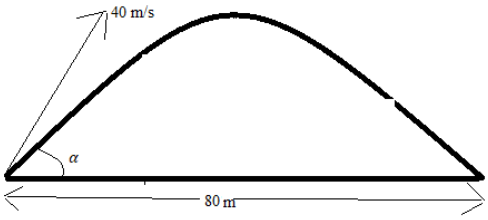
$$-5 = \left(\frac{21}{2}\right)t - \frac{9.8}{2}t^2 \quad \rightarrow 9.8t^2 - 21t - 10 = 0$$

$$t = 2.544s \ or \ t = -0.401s$$

Hence $t = 2.544s$

Horizontal displacement: $x = (21cos30)x2.544 = 46.2665m$

(b)



Time of flight: $T = \frac{2x40Sin\alpha}{10} \quad \rightarrow sin\alpha = \frac{T}{8} \ldots \ldots (i)$

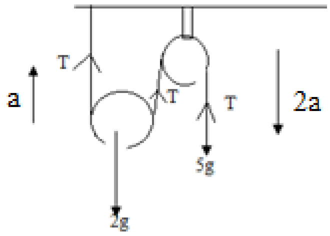
$$Range: 80 = (40cos\alpha)T \quad \rightarrow cos\alpha = \frac{2}{T} \ldots \ldots \ldots (ii)$$

$$(Sin\alpha)^2 + (cos\alpha)^2 = 1$$

$$\left(\frac{T}{8}\right)^2 + \left(\frac{2}{T}\right)^2 = 1$$

$$\frac{T^4 + 256}{64T^2} = 1 \quad \therefore T^4 - 64T^2 + 256 = 0$$

22.



(i) 2kg Pulley: $2T - 2g = 2a$…………(i)

5Kg mass: 5g – T = 5x2a ➡ T = 5(g – 2a)………….(ii)

Putting eqn(ii) in eqn(i)

2x5(g -2a) – 2g = 22a ➡ 22a=8g

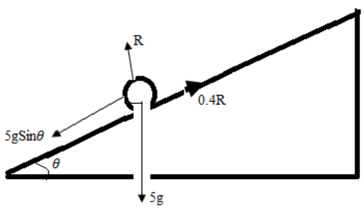
$\therefore a = \frac{8x9.8}{22} = \mathbf{3.5636m/s^2}$ $(acceleration\ of\ 2kg\ pulley)$ and

Acceleration of 5Kg mass = 2x3.5636 = **7.1272m/s²**

(ii) From eqn(ii); T =5(9.8 – 2x3.5636) = **13.364N**

(iii) $s = 0x1.5 + \frac{1}{2}x3.5636x1.5^2 = 4.0091m$

23. (a)



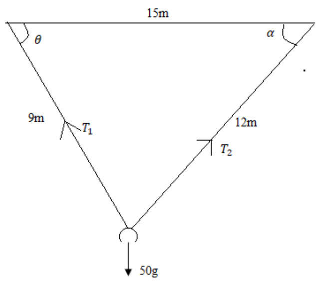
5gSin$\theta - 0.4R = 5a$, But $R = 5gCos\theta$

$$a = \frac{5x9.8(\frac{4}{5} - 0.4x\frac{3}{5})}{5} = 5.488m/s^2$$

From $v^2 = u^2 + 2as$

$$v^2 = 0 + 2x5.488x8 \quad \therefore v = 9.3906m/s$$

(b)



Resolving vertically: $T_1 Sin\theta + T_2 Sin\alpha = 50g$......................(i)

Resolving Horizontally: $T_1 Cos\theta = T_2 Cos\alpha$ ....(ii)

Using Cosine rule: $12^2 = 9^2 + 15^2 - 2x9x15Cos\theta$

$Cos\theta = \frac{9^2+15^2-12^2}{2x9x15} = \frac{3}{5}$ Hence $Sin\theta = \frac{4}{5}$ $\therefore \theta = 53.1°$

Also $9^2 = 12^2 + 15^2 - 2x12x15Cos\alpha$

$Cos\alpha = \frac{12^2+15^2-9^2}{2x12x15} = \frac{4}{5}$ Hence $Sin\alpha = \frac{3}{5}$ $\therefore \alpha = 36.9°$

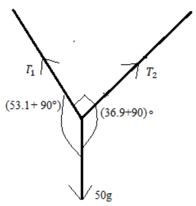
From (i) $T_1 x\frac{4}{5} + T_2 x\frac{3}{5} = 50g \longrightarrow$ $4T_1 + 3T_2 = 250g \ldots\ldots\ldots\ldots..(iii)$

(ii) $T_1 x\frac{3}{5} = T_2 x\frac{4}{5} \longrightarrow$ $T_1 = \frac{4}{3}T_2$ ....(iv)

Using (iv) in (iii) $4x\frac{4}{3}T_2 + 3T_2 = 250g$ $T_2 = \frac{3x250x9.8}{25} = 294\boldsymbol{N}$

Hence $T_1 = \frac{4}{3}x294 = \boldsymbol{392N}$

Method 2: Using Lami's theorem

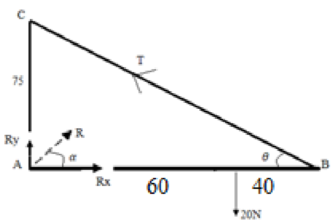


$$\frac{T_1}{Sin(36.1 + 90°)} = \frac{T_2}{Sin(53.1 + 90°)} = \frac{50g}{Sin90°}$$

$$\therefore T_1 = \frac{50x9.8xsin126.9°}{sin90°} = 391.8455 \approx 392N$$

Also; $T_2 = \frac{50x9.8xsin143.1°}{sin90°} = 294.2059 \approx 294N$

24.



Taking moments about point A;

$Tx100Sin\theta = 20x60$ But $Sin\theta = \frac{75}{125} = \frac{3}{5}$ and $Cos\theta = \frac{4}{5}$

$Tx100x\frac{3}{5} = 20x60$ $\therefore T = \frac{5x1200}{300} = 20N$

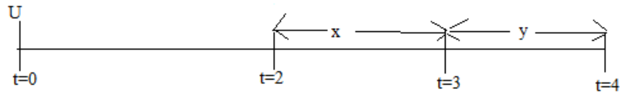
Resolving:

Vertically: $TSin\theta + R_y = 20 \rightarrow 20x\frac{3}{5} + R_y = 20 \rightarrow R_y = 8N$

Horizontally: $R_x = TCos\theta \rightarrow R_x = 20x\frac{4}{5} = 16N$

$Magnitude, |R| = \sqrt{8^2 + 16^2} = 17.8885N$ $Direction,$ $Tan\alpha = \frac{16}{8}$ $\alpha = 63.4°$

Hence the magnitude of the reaction at A is 17.8885N acting at angle of 63.4° to the horizontal.



25. (a)

Let U be the initial velocity.

$Using\ S = ut + \frac{1}{2}at^2$

$$\boldsymbol{In\ 3rd\ second}: x = \left(ux3 + \frac{1}{2}a(3)^2\right) - \left(ux2 + \frac{1}{2}a(2)^2\right) = u + \frac{5}{2}a$$

$$a = \frac{2}{5}(x-u) \dots\dots\dots (i)$$

$$\boldsymbol{In\ 4th\ second}: y = \left(ux4 + \frac{1}{2}a(4)^2\right) - \left(ux3 + \frac{1}{2}a(3)^2\right) = u + \frac{7}{2}a$$

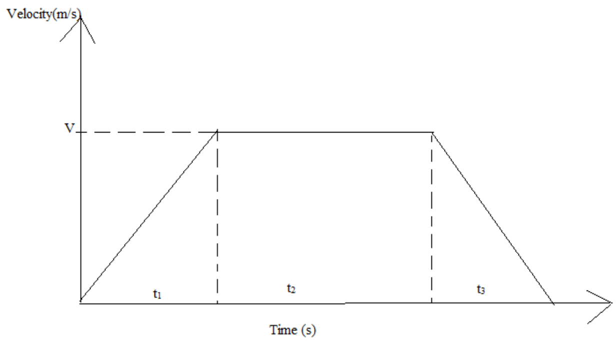
$$a = \frac{2}{7}(y-u) \dots\dots\dots (ii)$$

$Equating (i) and\ (ii)$

$\frac{2}{5}(x-u) = \frac{2}{7}(y-u) \quad \rightarrow x - u = \frac{5}{7}(y-u) \quad \rightarrow 7x - 7u = 5y - 5u$

$\rightarrow 2u = 7x - 5y \ \therefore \boldsymbol{u = \frac{1}{2}(7x - 5y)}$

(b).



$Acceleration\ region: x = \frac{1}{2}.V.t_1 \quad \rightarrow t_1 = \frac{2x}{V}$

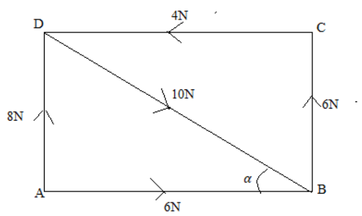
$Constant\ velocity\ region: 3x = V.t_2 \rightarrow t_2 = \frac{3x}{V}$

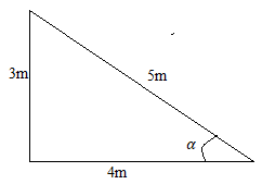
$Deceleration\ regio: 2x = \frac{1}{2}.V.t_3 \quad \rightarrow t_3 = \frac{4x}{V}$

$Total\ time: T = t_1 + t_2 + t_3$

$$T = \frac{2x}{V} + \frac{3x}{V} + \frac{4x}{V} = \boldsymbol{\frac{9x}{V}}s$$

26.

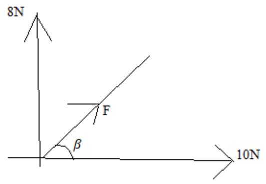




$Sin\alpha = \frac{3}{5}, Cos\alpha = \frac{4}{5}$

(i) Resultant force, $F = \binom{6}{0} + \binom{0}{6} + \binom{-4}{0} + \binom{0}{8} + \binom{10Cos\alpha}{-10Sin\alpha} = \begin{pmatrix} 2 + 10x\frac{4}{5} \\ 14 - 10x\frac{3}{5} \end{pmatrix} = \binom{10}{8}$

Magnitude $|F| = \sqrt{10^2 + 8^2} = \boldsymbol{12.8062N}$



Direction: $tan\beta = \frac{8}{10}$ $\quad \therefore \beta = \mathbf{38.7°}$

Hence the magnitude of resultant force is 12.8062N acting at angle of 38.7° to the horizontal.

(ii) Let the resultant force act at a point (x,y)

$$Resultant\ moment = \begin{vmatrix} x & y \\ 10 & 8 \end{vmatrix} = 8x - 10y \dots \dots \dots (i)$$

$$Taking\ moments\ about\ point\ B, M = 4x3 - 8x4 = -20Nm \dots \dots (ii)$$

$$Equating\ the\ moment\ of\ resultant\ force\ and\ sum\ of\ moment\ of\ individaul\ forces$$

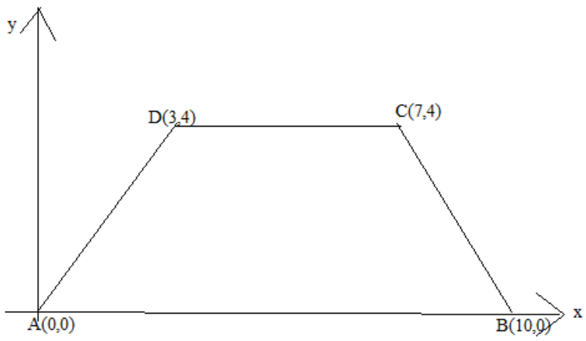
$$8x - 10y = -20$$

$$Hence\ \mathbf{4x - 5y + 10} = \mathbf{0}\ is\ the\ equation\ of\ the\ line\ of\ action\ of\ the\ resultant\ force$$

(iii) It cuts AB produced when y = 0, $4x - 5(0) + 10 = 0 \quad \rightarrow 4x = -10 \quad \rightarrow x = -2.5m$

Hence the line of action cuts AB at a distance of 2.5m from B on the left.

27. (a) $weight\ \propto lenght\ \ \rightarrow w\alpha\, l\ \ \rightarrow w = gl\ where\ g\ is\ constant$



$$AB = \sqrt{(10-0)^2 + (0-0)^2} = 10\ units$$

$$BC = \sqrt{(7-10)^2 + (4-0)^2} = 5\ units$$

$$CD = \sqrt{(7-3)^2 + (4-4)^2} = 4\ units$$

$$AD = \sqrt{(3-0)^2 + (4-0)^2} = 5\ units$$

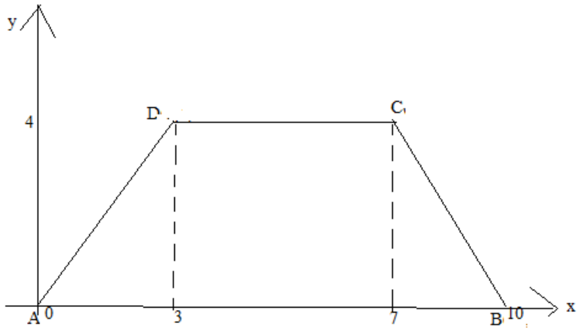
Note: Each length is uniform, so the resultant mass is at the centre

$$(10 + 5 + 4 + 5)g\begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix} = 10g\begin{pmatrix}5 \\ 0\end{pmatrix} + 5g\begin{pmatrix}8.5 \\ 2\end{pmatrix} + 4g\begin{pmatrix}5 \\ 4\end{pmatrix} + 5g\begin{pmatrix}1.5 \\ 2\end{pmatrix}$$

$$24\begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix} = \begin{pmatrix}120 \\ 36\end{pmatrix} \quad \therefore \begin{pmatrix}\bar{x} \\ \bar{y}\end{pmatrix} = \begin{pmatrix}5 \\ 1.5\end{pmatrix}$$

Hence centre of gravity is at a point (**5, 1.5**)

(b) (i) If ABCD is a lamina, $weight \propto area \rightarrow w\alpha A \rightarrow w = gA$

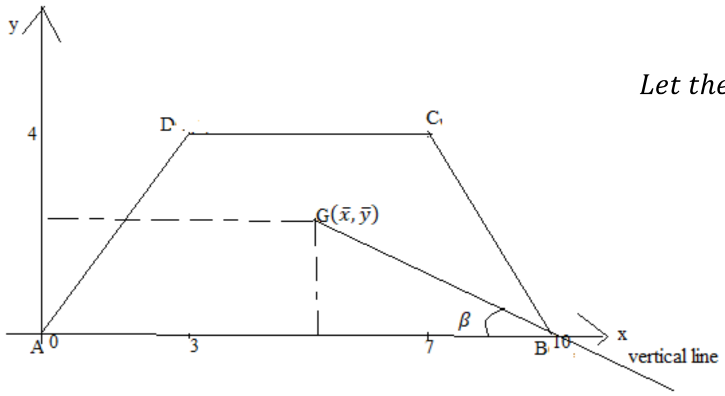


Body	Area (sq. units)	Weight	Position of c.o.g from origin
1st triangle	$\frac{1}{2}x3x4 = 6$	6g	$\left(\frac{2}{3}x3, \frac{1}{3}x4\right) = (2, \frac{4}{3})$
Rectangle	4x4 = 16	16g	$\left(3 + \frac{1}{2}x4, \frac{1}{2}x4\right) = (5,2)$
2nd triangle	$\frac{1}{2}x3x4 = 6$	6g	$\left(7 + \frac{1}{3}x3, \frac{1}{2}x4\right) = (8,2)$
Whole body	28	28g	$(\bar{x}, \bar{y})$

$$28g\binom{\bar{x}}{\bar{y}} = 6g\binom{2}{4/_3} + 16g\binom{5}{2} + 6g\binom{8}{2}$$

$$28\binom{\bar{x}}{\bar{y}} = \binom{140}{52} \quad \therefore \binom{\bar{x}}{\bar{y}} = \binom{5}{1.8571}$$

Hence centre of gravity is at a point (**5, 1.8571**)



Let the angle made by AB with vertical be β

$$Tan\beta = \frac{(10-5)}{1.8571} \quad \therefore \beta = 69.6°$$

28. (a) $F = ma \quad (2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}) = 5\boldsymbol{a} \quad \therefore \boldsymbol{a} = \frac{1}{5}(2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k})\boldsymbol{m/s^2}$

(b) $v = \int adt \quad \rightarrow v = \int \frac{1}{5}(2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k})\boldsymbol{dt} \quad v = \frac{1}{5}(2t\boldsymbol{i} + 3t\boldsymbol{j} - 4t\boldsymbol{k}) + \boldsymbol{c}$

$When\ t = 0, v = 0 \ \rightarrow c = 0$

$Hence$ when t=3s, $v = \frac{1}{5}(2x3\boldsymbol{i} + 3x3\boldsymbol{j} - 4x3\boldsymbol{k}) \ \rightarrow \boldsymbol{v} = \frac{\boldsymbol{1}}{\boldsymbol{5}}(\boldsymbol{6i} + \boldsymbol{9j} - \boldsymbol{12k})\boldsymbol{m/s}$

(c) $r(t) = r(0) + v.t \ \rightarrow r(3) = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 6/5 \\ 9/5 \\ -12/5 \end{pmatrix} x3 = \begin{pmatrix} 8/5 \\ 32/5 \\ -36/5 \end{pmatrix}$m

Distance from the origin $|r(3)| = \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{32}{5}\right)^2 + \left(\frac{-32}{5}\right)^2} \ = 9.1913m$

29. $\hat{V}c = \begin{pmatrix} 20sin40° \\ 20cos40° \end{pmatrix} \ \hat{V}w = \begin{pmatrix} 10cos60° \\ -10sin60° \end{pmatrix}$

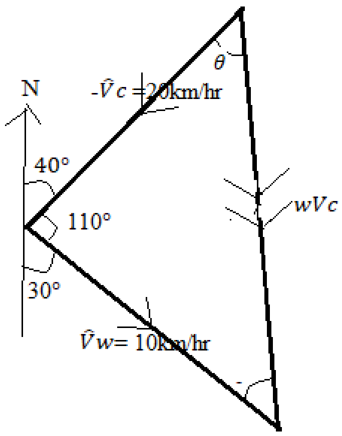
$wVc \ = \ Vw - Vc = \begin{pmatrix} 10cos60° \\ -10sin60° \end{pmatrix} - \begin{pmatrix} 20sin40° \\ 20cos40° \end{pmatrix} = \begin{pmatrix} -7.8558 \\ -23.9811 \end{pmatrix}$

$$|wVc| = \sqrt{(-7.8558)^2 + (-23.9811)^2} = 25.2350km/hr$$

Direction $tan\theta = \frac{23.9811}{7.8558} \ \therefore \theta = 71.9°$

Hence $|wVc| = 25.2350kmhr^{-1}\ acting\ at\ angle\ of\ 71.9°\ to\ the\ horizontal\ or$ $S18.1°\ W\ or 198.1°$

Method 2:



$$|wVc|^2 = 20^2 + 10^2 - 2x10x20Cos110°$$

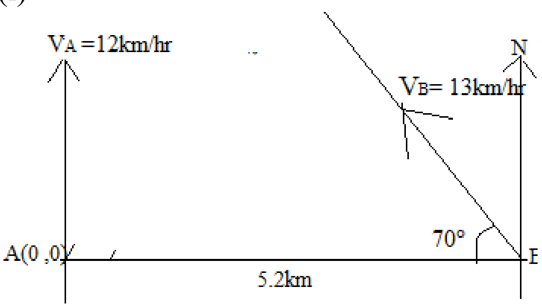
$$|wVc| = 25.2350\ km/hr$$

$$\frac{Sin\theta}{10} = \frac{Sin110°}{25.2350}$$

$$\theta = sin^{-1}\left(\frac{10Sin110°}{25.2350}\right) = 21.9°$$

$Direction\ is\ S18.1°\ W\ or\ 198.1°$

b(i)



$V_A = \begin{pmatrix} 0 \\ 12 \end{pmatrix}, \quad V_B = \begin{pmatrix} -13Cos70° \\ 13sin70° \end{pmatrix}$

$r_A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 12 \end{pmatrix} t = \begin{pmatrix} 0 \\ 12t \end{pmatrix}$

$r_B = \begin{pmatrix} 5.2 \\ 0 \end{pmatrix} + \begin{pmatrix} -13Cos70° \\ 13sin70° \end{pmatrix} t = \begin{pmatrix} 5.2 - 13tCos70° \\ 13tsin70° \end{pmatrix}$

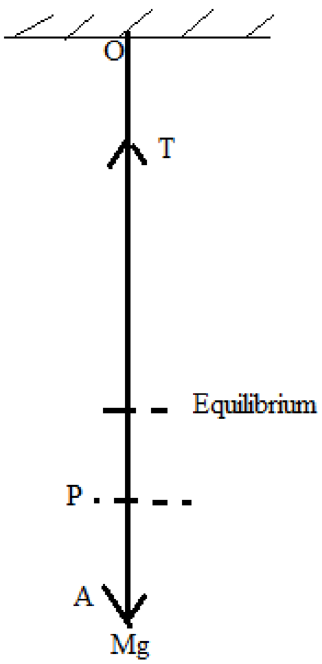
$_{B}r_{A} = r_B - r_A = \begin{pmatrix} 5.2 - 13tCos70° \\ 13tsin70° \end{pmatrix} - \begin{pmatrix} o \\ 12t \end{pmatrix} = \begin{pmatrix} 5.2 - (13Cos70°)t \\ (13sin70° - 12)t \end{pmatrix}$

(ii) At 8:30, $t = 30min - \frac{1}{2}hrs$

$_{B}r_{A}\left(t = \frac{1}{2}\right) = \begin{pmatrix} 5.2 - (13Cos70°)x0.5 \\ (13sin70° - 12)x0.5 \end{pmatrix} = \begin{pmatrix} 2.9769 \\ 0.1080 \end{pmatrix}$

$Distance\ between\ A\ and\ B = \sqrt{(2.9769)^2 + (0.1080)^2} = 2.9789km$

30. (i)



*Let **A** be the lowest point , **P** the general point and **y** the extension of the string.*

Using the principle of conservation of energy

$$(K.E + P.E + E.P.E)_o = (K.E + P.E + E.P.E)_A$$

$$(0 + mg(L + y) + 0) = 0 + 0 + \frac{1}{2}\frac{4Mg}{L}y^2$$

$$(L + y) = \frac{2}{L}y^2$$

$$2y^2 - Ly - L^2 = 0$$

$$(2y9 + L)(y - L) = 0$$

$$y = -\frac{L}{2}\ or\ y = L$$

$Hence\ greatest\ distance\ below\ O = \boldsymbol{L + L = 2L}$

Let the extension at equilibrium be $\boldsymbol{e}$

At equilibrium: $\boldsymbol{Mg = T = \left(\frac{4Mg}{L}\right)e \quad \rightarrow e = \frac{L}{4}}$

When displaced through a distance, $\boldsymbol{x}$, restoring force,

$$\boldsymbol{Force = Mg - T^1, where\ T^1\ is\ new\ tension}$$

$$\boldsymbol{Ma = Mg - \frac{4Mg}{L}(x + e)}$$

$$\boldsymbol{Ma = Mg - \frac{4Mg}{L}x - \frac{4Mg}{L}e; \quad But: Mg = \left(\frac{4Mg}{l}\right)e}$$

$$\boldsymbol{Ma = Mg - \left(\frac{4Mg}{L}\right)e - Mg = -\left(\frac{4Mg}{L}\right)e}$$

$$\therefore a = -\left(\frac{4g}{L}\right)x$$

Compare with $a = -\omega^2 x \quad \rightarrow \omega^2 = \frac{4g}{L} \quad \rightarrow \omega = \sqrt{\frac{4g}{L}} = 2\sqrt{\frac{g}{L}}$

$$Amplitude, A = L - \frac{L}{4} = \frac{3L}{4}$$

$$Acceleration\ at\ L = A\omega^2 = \frac{3L}{4} x \frac{4g}{L} = 3g$$

(ii) $When\ acceleration\ is\ zero, velocity\ is\ maximum,$

$$V_{max} = Aw = \frac{3L}{4} x2\sqrt{\frac{g}{L}} = \frac{3}{2}\sqrt{gL}$$

Hence speed is $1.5\sqrt{gL}$

NUMERICAL METHODS

31. (a) (i)

48.11	x	46.93
7.621	8.614	9.043

$$\frac{9.043-7.621}{46.93-48.11}=\frac{8.614-7.621}{x-48.11}$$

$$\therefore x = 47.2861$$

(ii)

51.07	50.24	48.11
y	4.116	7.621

$$\frac{7.621-y}{48.11-51.07}=\frac{7.621-4.116}{48.11-50.24} \qquad \therefore y = 2.7503$$

$$Hence\ f^{-1}(51.07) = 2.7503$$

(b)

$h=\frac{2-1}{5}=0.2$ Let $y=\frac{x^2}{x^2+1}$

x	y_0, y_6	$y_1,\dots y_5$
1.0	0.5	
1.2		0.5902
1.4		0.6622
1.6		0.7191
1.8		0.7642
2.0	0.8	
Sum	**1.3**	**2.7357**

$$\int_1^2 \frac{x^2}{x^2+1}dx \approx \frac{1}{2}x0.2(1.3+2x2.7357)=0.677\ \ (3d.ps)$$

(c) $\int_1^2 \frac{x^2}{x^2+1}dx=\int_1^2\left(1-\frac{1}{x^2+1}\right)dx=[x-tan^{-1}(x)]_1^2=$

$$=\left(2-tan^{-1}(2)\right)-\left(1-tan^{-1}(1)\right)==0.678$$

Error can be reduced by increasing the number of strips or sub-intervals

32. $h=\frac{2-1}{6}=\frac{1}{6}$ Let $y=(x-1)lnx$

x	y_0, y_6	$y_1,\dots y_5$
1.0	0.00000	
$^7/_6$		0.02569
$^4/_3$		0.09589
$^3/_2$		0.20273
$^5/_3$		0.34055
$^{11}/_6$		0.50511
2.0	0.69315	
Sum	**0.69315**	**1.16997**

$$\int_1^2 (x-1)lnxdx \approx \frac{1}{2}x\frac{1}{6}(0.69315+2x1.16997) \qquad =0.2528\ \ (4d.ps)$$

Exact value: $\int_1^2 (x-1)lnxdx=\frac{1}{2}[lnx(x^2-2x)]_1^2-\int_1^2\left(\frac{x^2}{2}-x\right).\frac{1}{x}dx$

$$=\frac{1}{2}[lnx(x^2-2x)]_1^2-\int_1^2\left(\frac{x}{2}-1\right)dx$$

$$=\frac{1}{2}\left[\left(lnx(x^2-2x)\right)\right]_1^2-\left[\frac{x^2}{4}-x\right]_1^2=(0-0)-\left(-1--\frac{3}{4}\right)=0.25$$

$$Maximum\ error\ =|Exact\ value-approx.value|=|0.25-0.2528|=0.0028$$

33. Error, $e = (x + \Delta x)^2(y + \Delta y) - x^2 y$

$= (x^2 + 2x\Delta x + (\Delta x)^2)(y + \Delta y) - x^2 y$

$= x^2 y + 2xy\Delta x + y(\Delta x)^2 + x^2\Delta y + 2x\Delta y\Delta x + \Delta y(\Delta x)^2 - x^2 y$

$= 2xy\Delta x + y(\Delta x)^2 + x^2\Delta y + 2x\Delta y\Delta x + \Delta y(\Delta x)^2$

Assumption: If $\Delta x \ll x$ and $\Delta y \ll y, then$ $(\Delta x)^2 \approx \Delta y\Delta x \approx \Delta y(\Delta x)^2 \approx 0$,then;

$e = 2xy\Delta x + x^2\Delta y$

Absolute error, $|e| = |2xy\Delta x + x^2\Delta y|$

$|e| \leq |2xy\Delta x| + |x^2\Delta y|$

Hence maximum error, $|\boldsymbol{e}| = |\boldsymbol{2xy\Delta x}| + |\boldsymbol{x^2\Delta y}|$

$|e| = |2(2.8)(1.44)0.0016| + |2.8^2(0.008)| = 0.0756$

Upper limit = $2.8^2 x 1.44 + 0.0756$ = 11.3652

Lower limit = $2.8^2 x 1.44 - 0.0756$ = 11.2140

34. (a) Let $f(x) = 3x^3 - x + 5$

$f(-1.4) = 3(-1.4)^3 - (-1.4) + 5 = -1.832$

$f(-1.2) = 3(-1.2)^3 - (-1.2) + 5 = 1.016$

Since $f(-1.4) < 0$ and $f(-1.2) > 0, then - 1.4 < x_{root} < -1.2$

(b)

-1.4	x_o	-1.2
-1.832	0	1.016

$\frac{1.016--1.832}{-1.2--1.4} = \frac{1.016-0}{x_o--1.4}$ $\quad \therefore x_o = -1.329$

(c) $f(x) = 3x^3 - x + 5 \quad \rightarrow f^1(x) = 9x^2 - 1$

$x_{n+1} = x_n - \frac{f(x)}{f^1(x)} = x_n - \frac{(3x^3-x+5)}{9x^2-1}$

$x_1 = -1.329 - \frac{(3(-1.329)^3-(-1.329)+5)}{9(-1.329)^2-1} = -1.281$

$x_2 = -1.281 - \frac{(3(-1.281)^3-(-1.281)+5)}{9(-1.281)^2-1} = -1.279$

$|e| = |-1.279 - -1.281| = 0.002 < 0.005$

Hence $\boldsymbol{x_{root} \approx -1.28}$

35. (a) (i) $\Delta l = 0.5,\ \Delta w = 0.005$

(ii) $Maximum\ area = 8.5x4.265 = 36.25$

$Minimum\ area = 7.5x4.255 = 31.91$

$Range = [31.91,\quad 36.25]$

(b) $volume\ of\ a\ cylinder, v = \frac{1}{3}r^2h$

Error in volume, $\Delta v = \frac{1}{3}(r+\Delta r)^2(h+\Delta h) - \frac{1}{3}r^2h$

$$= \frac{1}{3}(r^2 + 2r\Delta r + (\Delta r)^2).(h+\Delta h) - \frac{1}{3}r^2h$$

$$= \frac{1}{3}[(r^2h + 2rh\Delta r + (\Delta r)^2h) + r^2\Delta h + 2r\Delta r\Delta h + (\Delta r)^2\Delta h - r^2h]$$

$$= \frac{1}{3}[(2rh\Delta r + (\Delta r)^2h) + r^2\Delta h + 2r\Delta r\Delta h + (\Delta r)^2\Delta h]$$

$Assumptions: If\ \Delta r \ll r\ and\ \Delta h \ll h, then (\Delta r)^2 \approx \Delta r\Delta h \approx (\Delta r)^2\Delta h \approx 0$

$$\Delta v = \frac{1}{3}[2rh\Delta r + r^2\Delta h]$$

$$Relative\ error: \frac{\Delta v}{v} = \frac{\frac{1}{3}[2rh\Delta r + r^2\Delta h]}{\frac{1}{3}r^2h} = 2\frac{\Delta r}{r} + \frac{\Delta h}{h}$$

Absolute relative error $\left|\frac{\Delta v}{v}\right| = \left|2\frac{\Delta r}{r} + \frac{\Delta h}{h}\right|$

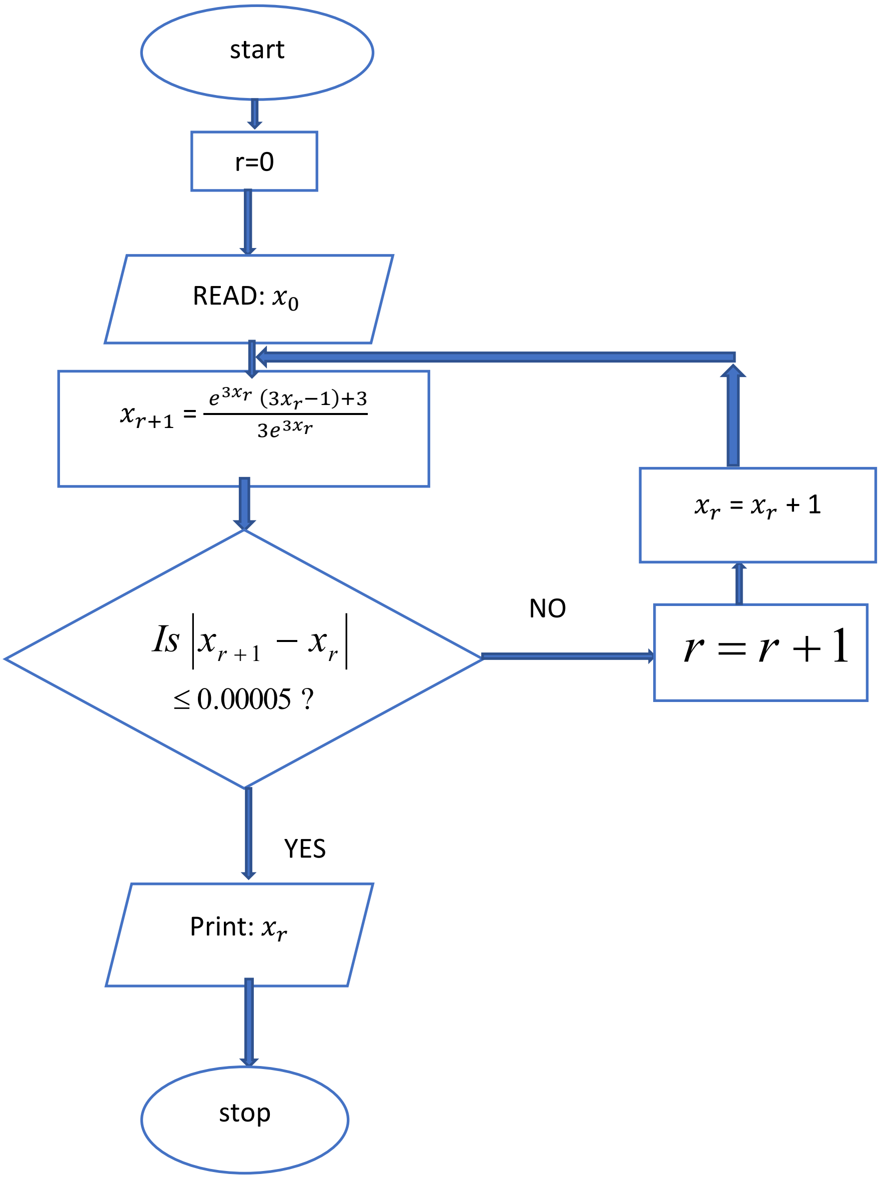
$$\left|\frac{\Delta v}{v}\right| \le \left|2\frac{\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right|$$

$$\boldsymbol{Maximum\ possible\ error} = \left|\frac{\boldsymbol{\Delta h}}{\boldsymbol{h}}\right| + \mathbf{2}\left|\frac{\boldsymbol{\Delta r}}{\boldsymbol{r}}\right|$$

36. (a) $3x = ln3$ $\quad e^{3X}$ =3

$let\ f(x) = e^{3X}$-3 $\quad$ f^{1}(x)=3e^{3X}

$$X_{r+1} = X_r - \left(\frac{e^{3X_r-3}}{3e^{3x_r}}\right) = \frac{3x_r e^{3x_r} - e^{3x_r} + 3}{3e^{3x_r}} = \frac{1}{3}\left[\frac{e^{3x_r}(3x_r-1)+3}{e^{x_r}}\right]$$



r	x_0	x_{r+1}	$\lvert x_{r+1}$ - $x_r\rvert$
0	0.33333	0.36788	0.03455
1	0.36788	0.36621	0.00167
2	0.36621	0.36620	0.00001

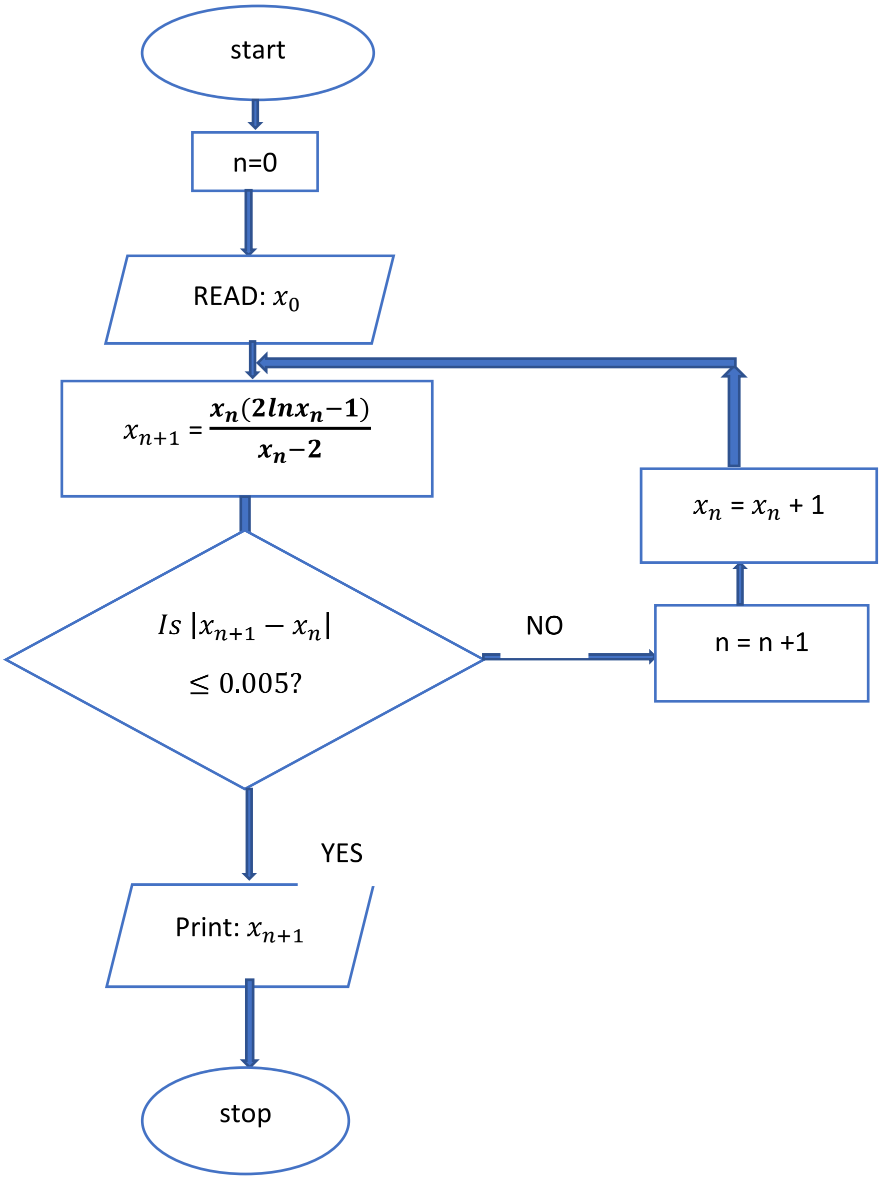
Hence $\boldsymbol{x_{root}}$ = **0.3662**

37. (a) $Let\ f(x) = x - 1 - 2lnx \quad \rightarrow \quad f^1(x) = 1 - \frac{2}{x}$

$x_{n+1} = x_n - \left(\frac{x_n - 1 - 2lnx_n}{1 - \frac{2}{x_n}}\right) = \frac{x_n - 2 - (x_n - 1 - 2lnx_n)}{\frac{x_n - 2}{x_n}} = \frac{x_n^2 - 2x_n - (x_n^2 - x_n - 2x_n lnx_n)}{x_n - 2}$

$$= \frac{x_n^2 - 2x_n - x_n^2 + x_n + 2x_n lnx_n}{x_n - 2}$$

$$= \frac{\boldsymbol{x_n(2lnx_n - 1)}}{\boldsymbol{x_n - 2}}$$



END

NABISUNSA GIRL'S SCHOOL.

A – LEVEL MATHEMATICS SEMINAR 2024.

P425/1 PURE MATHEMATICS.

PAPER STRUCTURE

	SECTION A	SECTION B
TRIGONOMETRY	1	1
VECTORS	1	1
GEOMETRY	1	1
ALGEBRA	2	2
ANALYSIS (CALCULUS)	3	3

GEOMETRY	ALGEBRA	ANALYSIS
• Coordinate geometry • Circles and locus • Parabolas • Ellipse • Hyperbola	• Permutations and combination • QEs and polynomials. • logarithms and indices. • Partial fractions • Series • complex numbers. • Inequalities • equations • Ratio theorem. • binomial and pascals triangle.	• Integration • Differentiation • Curve sketching, I and II • Differential equations • Inequalities • Rates of change • Small changes • Maclaurin's theorem

TRIGONOMETRY.

1. Solve the equation $\left[cos\left(\frac{x}{2}\right)+sin\frac{x}{2}\right]^2=\frac{3}{2}$ for $0^0 \leq x \leq 360^0$.
2. Solve the equation $sin2x+sin3x+sin4x=0$ for $-90 \leq x \leq 90$.
3. Solve the equation: $5sin2x-10sin^2x+4=0$ for $-\pi \leq x \leq \pi$.
4. Solve; $4sin^2x-12sin2x+35cos^2x=0$ for $0 \leq x \leq 90$.
5. Solve the equation. $5cos^2 3x=3(1+sin3x)$ fo $0 \leq x \leq 90$.
6. Solve $8cos^4x-10cos^2x=0$ for $0 \leq x \leq 2\pi$.
7. solve the equation.$tan^2x-sin^2x=1$ for $0^0 \leq x \leq 360$.
8. solve the equation: $tan^{-1}(2x+1)+tan^{-1}(2x-1)=tan^{-1}12$
9. *Find the maximum and minimum values of* $\frac{1}{6+4sinx-3cosx}$
10. *Express* $5sin^2x-3cosxsinx+cos^2x$. in form of $a+bcos(2x-B)$ *and hence find the minimum and maximum values of the expression.*
11. *Given that* $cosA=\frac{3}{5}, cosB=\frac{12}{13}$ *where A is obtuse and B is acute, find the exact values* of $\tan(A+B), cosec(A-B)$ *without using calculators* or *tables.*
12. *Express* $\sqrt{\frac{1-sin2x}{1+sin2x}}$ *in terms of tanx.*
13. If $tan^{-1}a+tan^{-1}b+tan^{-1}c=\pi$, *show that* $a+b+c=abc$.
14. If $sin3x=p$, $sin^2x=\frac{3}{4}-q$, *prove that* $p^2+16q^3=12q^2$.
15. *Eliminate* θ *between the equations* $x=acos^2\theta+bcos^2\theta$ and $y=(a-b)sin\theta cos\theta$.
16. *In triangle,* $s-a=3cm$, $s-b=4cm$, $s-c=5cm$ *where S is semi perimeter, find the area of triangle.*

Prove the following identities.

17. $4cos3xcosx+1=\frac{sin5x}{sinx}$.
18. $\frac{sinx+sin3x+sin5x}{cosx+cos3x+cos5x}=tan3x$.
19. $\frac{sinx+2sin2x+sin3x}{sinx-2sin2x+sin3x}=cot^2\left(\frac{x}{2}\right)$.
20. $Cos2A+cos2B+cos2C=-1-4cosAcosBcosC$.
21. $\frac{\sin(A+B)}{\cos(A-B)}+1=\frac{(1+cotA)(1+tanB)}{cotA+tanB}$.

VECTORS.

22. Vectors $\mathbf{a} = 2i - 2j - 2k$ and $\mathbf{b} = i - 3j + 2k$ form two sides of the triangle, find its area.

23. Show that $A(4,10,6)B(6,8,-2)$ and $C(1,10,3)$ are vertices of a right angled triangle.

24. Show that A (4, -8, -13), B (3, -2, -3), C (3,1, -2) are vertices of the triangle.

25. The point R divides the line AB externallyin the ratio of $3:1$ with $A(4,-3)$ and $B(-6,a)$. find coordinates of R and state the ratio in which point B divides AR.

26. Given the points $A(3,-2,5), B(9,1,-1)$. Find the coordinates of C such that C divides AB in the ratio of $5:3$. (i) externally (ii) internally.

27. The line passes through the points $A(4,6,3)$ and $B(1,3,3)$.
(a) find the vector equation of the line containing such points and hence show that a point $C(2,4,3)$ lies on the line.

28. Given that $a = i + 2j + 3k$ and $b = 4i - j + 2k$ are vectors, find a vector which is perpendicular to both a and b.

29. find the equation of the line through $A(1,-2,3)$ perpendicular to line $\frac{x-5}{2} = y - 2 = \frac{z-1}{3}$.

30. The points $A(2,3,-4), B(5,-1,2), C(11,\alpha,14)$.(
(i) find the unit vector parallel to AB.
(ii) find the position vector of D such that $ABCD$ is a parallelogram.

31. Given that $P(4,-3,5), Q(1,0,2)$. Find the coordinates of R such that $PR:PQ = 1:2$ and P, Q, R are collinear.

32. find the equation of a plane containing the line whose equation $r = (t-1)i + (t+2)j + (2t-4)k$ which is parallel to the direction vector $2i + 3j + k$, hence state the distance from the origin to this plane.

33. Given that the vectors $r = \begin{pmatrix} 5 \\ 3 \\ -5 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $r = \frac{x-5}{-1} = \frac{2-y}{3} = \frac{z+4}{4}$ intersect,
(i) find theposition vector of the point of intersection.
(ii) Cartesian equation of the line passingthrough the point of intersection of the lines above and parallel to the line $x = \frac{y-2}{2} = \frac{z-2}{3}$.

34. Show that the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{2-y}{-1} = \frac{2-z}{-1}$ don't intersect, hence find the shortest distance between them.

35. Plane passes through the point $(1,2,3)$ and is perpendicular to the vector $i - 5j + 4k$. The plane meets $z-$plane in P and the $y-$plane in Q, find the equation of the plane and the distance PQ.

36. A line through the point $D(-13,1,2)$ and parallel to the vector $12i + 6j + 3k$ meets the plane containing the lines $r = (-2i + 5j - 11k) + \beta(3i + j + 2k)$ and $r = 8i + 9j + \alpha(4i + 2j + 5k)$ at E.
find the coordinates of E and the angle between the line and the plane.

37. Determine the equation of the plane passing through the point $P(1,2,3)$ and parallel to the lines $r = 3i + 3j - k + \alpha(i - j - k)$ and $r = 4i - 5j - 8k + \beta(3i + j - 2k)$.

38. Find the line of intersection of the planes $2x + 3y + 4z = 1$ and $x + y + 3z = 0$.

39. Find the orthocentre of the triangle $A(-2,1), B(3,-4)$ $C(-6,-1)$. (b) a point $C(a,4,5)$ divide the line joining $A(1,2,3)$ and $B(6,7,8)$ in the ratio of $\alpha:3$. Find the values of a and α.

40. A plane contains points $A(4,-6,5)$ and $B(2,0,1)$.
A perpendicular to the plane from $P(0,4,-7)$ intersect the plane at C.
find the Cartesian equation of the line PC.

41. The point $(6,-9,5)$ lies on the line $\frac{x-a}{3} = \frac{y-5}{b} = \frac{z-c}{-4}$ which is parallel to the plane $3x + y + 4z = 3$.
Find the values of a, b, c and the shortest distance between the line and the plan

42. In the triangle OAB has $OA = a, OB = b$, C is a point on OA such that $OC = 2/3a$. D is a midpoint of AB, when CD is produced it meets OB produced at E such that $DE = nCD$ and $BE = kb$.
Express DE in terms of (I) n, a, b (ii) k, a, b and hence find the values of n and k

43. Given the equation of two lines $y = m_1x + c_1$ and $y = m_2x + c_2$, show that the vector equations are $\begin{pmatrix}0\\c_1\end{pmatrix} + \mu\begin{pmatrix}1\\m_1\end{pmatrix}$ and $\begin{pmatrix}0\\c_2\end{pmatrix} + \alpha\begin{pmatrix}1\\m_2\end{pmatrix}$.
Hence show that the angle between the lines is $\tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1m_2}\right)$.

GEOMETRY.

44. $A(-3,0)$ *and* $B(3,0)$ *are fixed points.* Show that the locus of $P(x,y)$ which moves such that $PB = 2PA$ *is a circle and find its radius and circle.*

45. *Given that* $r = 3\cos\theta$ *is an equation of a circle. Find its Cartesian form.*

46. The point $A(x,1)$ and $B(-6,-5)$ are equidistant from the point $C(3,-2)$. Find the value of x.

47. The line $y = mx$ intersects the curve $y = 2x^2 - x$ at the points A &B. Find the *equation of locus of point P which divides AB in the ratio* $2:5$.

48. *Find the* equation *of the circumscribing circle which passés* through *the points* $(1,2), (2,5)$ *and* $(-3,4)$.

49. *Find the parametric equation of the circles* $(x+1)^2 + (y-2)^2 = 9$

50. *Find the length of the tangent to the circle from the point* $(5,7)$ to *the circle* $x^2 + y^2 - 4x - 6y + 9 = 0$.

51. *Show that* $y = mx + c$ *is a tangent to the circle* $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$.

52. *Show that the line* $y = x + 1$ *touches the circle* $x^2 + y^2 - 8x - 2y + 9 = 0$ *hence find the point(s) of intersection.*

53. *Prove that the circle* $x^2 + y^2 - 2x - 6y + 1 = 0$ and $x^2 + y^2 - 8x - 8y + 31 = 0$ in two
distinct places and find the equation of the common chord.

54. *Show that the tangents of* $x^2 + y^2 + 4x - 2y - 11 = 0$ and $x^2 + y^2 - 4x - 8y + 11 = 0$ *are intersecting at right angles.*

55. *Find the equation of a circle whose center lies on the line* $y = 3x - 1$ *and passes through the points* $(1,1)$ and $(2,-1)$.

56. *Sketch the parabola* $y^2 + 8y - 4x + 12 = 0$ *showing clearly the focus and the directrix.*

57. *Show that the parametric equations* $x = 3t^2 - 2,\ y = -6t$ *represent the* parabola. *Find the focus and the directrix and hence sketch it.*

58. *If the normal* at $P(ap^2, 2ap)$ *to the parabola* $y^2 = 4ax$ *meets the curve again at* $Q(aq^2, 2aq)$. *Prove that* $p^2 + pq + 2 = 0$.

59. *Show that the that the tangent drawn from the end points* of *the focal chord joining the points* $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ *intersect at* 90^0 *at the directrix.*

60. *Prove that the chord* $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ *on the parabola* $y^2 = 4ax$ *has the equation* $(p+q)y = 2x + 2apq$.
(b) *a variable chord PQ of the parabola is*
such that the line OP and OQ are perpendicular, where O is the origin.

(i)$prove\ that\ the\ chord\ PQ\ cuts\ the\ x-axis$ at $the\ fixed\ points, give\ the\ x-coordinate$ of the point. (ii) $find\ the\ equation\ of\ the\ locus\ of\ the\ midpoint\ PQ.$

61.$If\ the\ line\ y = mx + c\ is\ a\ tangent\ to\ the\ ellipse\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when $c \pm \sqrt{a^2m^2 + b^2}$. $Hence\ find\ the\ equation\ of\ tangents\ to\ the\ ellipse\ \frac{x^2}{4} + \frac{y^2}{1} = 1\ from\ the\ point\ (0, \sqrt{5})$.

62.$Given\ that\ y = mx + c\ is\ a\ tangent\ to\ the\ hyperbola\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $show\ that\ c^2 = a^2m^2 - b^2$.

63.$Show\ that\ 25x^2 + 9y^2 - 100x - 54y = 44\ represents\ an\ ellipse$, state the $coordinates\ of\ the\ center\ and\ the\ eccentricity$.

64.$The\ normal\ to\ the\ parabola\ y^2 = 4ax$ at the point $P(at^2, 2at)$ meets the axis of $the\ parabola\ at\ G\ and\ GP\ is\ produced\ beyond\ P\ to\ Q\ so$ that $GP = PQ$, show $that\ the\ locus\ of\ Q\ is\ given\ by\ y^2 = 16a(x + 2a)$.

65.$Prove\ that\ the\ equation\ of\ the\ chord\ joining\ the\ points\ P(cp, c/p)$ and $Q(cq, c/q)\ on\ the\ rectangular\ hyperbola\ xy = c^2$ is $x + pqy = c(p + q)$.
(ii) $if\ this\ chord\ is\ also\ normal\ at\ P$, Show that $p^3q + 1 = 0$.
$If\ in\ this\ case\ the\ normal$ at $Q\ cuts\ the\ hyperbola\ again\ at\ R. prove\ that$ PR $has\ the\ equation\ x + py = cp(1 + p)$.

ALGEBRA.

66.$Use\ the\ substitution\ x^2 - 4x = y$ to solve $2x^4 - 16x^3 + 77x^2 - 180x + 63 = 0$.

67.$Find\ the\ square\ root$ of $6 + 14\sqrt{5}$.

68.$Solve\ the\ equation\ \sqrt{(2x+3)} - \sqrt{(x+1)} = \sqrt{x-2}$.

69.$Solve\ the\ equation$: $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

70.If $a^2 + b^2 = 7ab$, show that $\log \frac{1}{3}(a + b) = \frac{1}{2}(\log a + \log b)$

71.$Solve\ the\ equation$: $x^2 + 2x + \frac{12}{x^2+2x} = 7$

72.$Solve\ the\ equation$: $x^{\frac{4}{3}} + 16x^{\frac{-4}{3}} = 17$.

73.$Solve$: $(0.4)^{-3x} < 3.6$.

74.$Solve\ the\ inequality$: $\frac{x+1}{2x-1} \leq \frac{1}{x-3}$.

75.$Prove\ by\ induction\ that\ 2^{4n} - 1\ is\ a\ multiple\ of\ 15$.

76.$Prove\ by\ induction\ that\ 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$ $and\ deduce\ that\ (n+1)^3 + (n+2)^3 + \cdots + (2n)^3 = \frac{1}{4}n^2(3n+1)(5n+3)$.

77. *The sum of the first* n *– terms of a certain progression is* $n^2 + 5n$ for all *integral value of* n. *find the first three terms and prove that* the progression *is an AP*.

78. *The second, fourth and eighth term of An AP are in GP.*
If the sum of the third and the fifth term is 20.
Find the sum of the first 4 *terms of the progression.*

79. *Expand* $\sqrt{\frac{1+5x}{1-5x}}$ *as far as the term including* x^3.
Taking the first three terms, evaluate $\sqrt{14}$ *to* 3*sf*.

80. *Given that* $Z = 1$ and $Z = 1 + i$ *are roots of the equation* $Z^3 + aZ^2 + bZ + c = 0$. *Find the values of* a, b *and* c.

81. *Given that the complex number* Z *varies such that* $|Z - 7| = 3$.
Find the greatest and least value of $|Z - i|$.

82. *Find the locus defined by* $|Z - 2 + 3i| \geq 2$ *if* Z *is a complex number.*

83. *Show that the locus of* $\arg\left(\frac{Z-1}{Z-i}\right) = \frac{\pi}{6}$ *is a circle,*
find its centre and radius.

84. *Using Demoivre's theorem,* prove that $16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$.

85. *Given the complex number* Z, $Z = \frac{(3i+1)(i-2)^2}{i-3}$, *determine the modulus* and *argument of* Z.

86. *Prove that if* $\frac{Z-6i}{Z+8}$ *is real, then the locus of the point representing the complex number* Z *is a straight line.*

87. If Z_1 and Z_2 *are complex number, solve the simultaneous equations.*
$4Z_1 + 3Z_2 = 23$ and $Z_1 + iZ_2 = 6 + i8$ *in* $x + iy$.

88. *Simplify.* $\frac{(\cos\frac{2\pi}{7} - i\sin\frac{2\pi}{7})^3}{(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7})^4}$

89. *Show that if the equation* $x^2 + ax + 1 = 0$ and $x^2 + x + b = 0$ *have common* roots, then $(b-1)^2 = (a-1)(1-ab)$.

90. *When the polynomial* $3x^3 + ax^2 - bx + 1$ *is divided by* $(x-2)^2$ *the remainder* is $39x - 51$. *Find the values of* a *and* b.

91. *Given that the equation* $y^3 - 2y + 4 = 0$ and the $y^2 + y + c = 0$ *have a common root, show that* $c^3 + 4c^2 + 14c + 20 = 0$.

92. *If* α^2 *and* β^2 *are roots of the equation* $x^2 - 21x + 4 = 0$.
Form an equation with roots α *and* β.

93. *A polynomial* $p(x)$ *is a multiple of* $x - 3$
and the remainder when divided by $x + 3$ is 12.
Find the remainder when the polynomial is divided by $x^2 - 9$.

ANALYSIS

94. $find\ the\ area\ bounded\ by\ the\ curve\ y = (5 - x)(x + 1)\ the\ y-axis\ and\ the\ line\ y = 5.$

95. $Prove\ that\ the\ area\ enclosed\ by\ the\ two\ parabolas\ y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. $If\ this\ area\ is\ rotated\ through\ four\ right\ angles\ about\ the\ x-axis, show\ that\ the\ volume\ generated$ is $\frac{96}{5}\pi a^3$.

96. $Show\ that\ \int_e^{e^2} \frac{dx}{x\ln x} = \ln 2.$

$Integrate\ the\ following.$

97. $\int \frac{1-(log10^x)^{-1}}{x\ln 10} dx.$

98. $\int \frac{x^3 dx}{1+x^8}$

99. $\int \frac{dx}{3\sin^2 x+\cos^2 x}.$

100. $\int \frac{dx}{1+\sin x+\cos x}.$

101. $\int \sin 4x \cos 2x dx.$

102. $\int \sqrt{\frac{3+x}{3-x}} dx$, using $x = 3\sin\theta$

103. $Show\ that\ \int_0^1 \frac{8x+6}{(x^2+1)(x+2)} dx = \pi + \ln\frac{8}{9}.$

104. $Differentiate\ \cos 2x\ from\ first\ principles.$

105. $If\ y = 4^x \sin x, find\ the\ value$ of $\frac{dy}{dx}$

106. $Given\ that\ y = ae^{-2x}\sin 3x, prove\ that\ \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0.$

107. If $y = Ae^{3x} + Be^{-2x}$. $show\ that\ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$

108. $Differentiate$: $\sin(x^x)$, $2x^x$ and $2x^{\cos x}$.

109. $If\ x = \frac{3t-1}{t}\ and\ y = \frac{t^2+4}{t}, show\ that\ \frac{d^2y}{dx^2} = 2t^3.$

110. $If\ y = \sqrt{\frac{1+\sin x}{1-\sin x}}, show\ that\ \frac{dy}{dx} = \frac{1}{1-\sin x}$

111. $Differentiate: \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).$

112. $Given\ that\ y = \tan xy, show\ that\ \frac{dy}{dx} = \frac{y}{\cos^2 xy-x}$

113. $Differentiate: y = \ln\sqrt{\frac{1+x}{1-x}}.$

114. $Use\ small\ changes\ to\ evaluate\ \sin 48^0$

115. $Use\ small\ changes\ to\ evaluate\ \sqrt[3]{30}.$

116. $A\ cylinder\ of\ radius\ r\ and\ height\ 8r. The\ radius\ increases\ from$ 4cm to 4.1cm. $find\ the\ approximate\ increase\ in\ volume.$

117. $The\ length\ of\ a\ rectangular\ block\ is\ three\ times\ its\ width.$ $The\ total\ surface\ area\ of\ the\ block$ is 180cm^2. $Find\ its\ maximum\ value.$

118. $A\ piece\ of\ wire\ of\ length\ L\ is\ cut\ into\ two\ portions\ of\ length\ x$ and $l-x$.
$Each\ piece\ is\ the\ cut\ into\ twelve\ equal\ parts\ soldered\ together\ so\ as\ to$ $form\ the\ edges\ of\ the\ cube. Show\ that\ the\ volume\ is\ given$ by $V = \frac{l^3-3l^2x+3lx^2}{1728}$ $and\ that\ the\ minimum\ volume\ is$ $\frac{l^3}{6912}$

119. $The\ base\ radius\ of\ a\ circular\ cone\ increases\ and\ the\ volume\ changes$ by 2%. $If\ the\ height\ of\ the\ cone\ remains\ constant,$ $find\ the\ percentage\ increase\ in\ the\ circumference\ of\ the\ base.$

120. $A\ capsule\ consists\ of\ a\ cylinder\ and\ two\ identical\ hemispheres\ as\ shown.$ $Show\ that\ the\ surface\ area\ S = \frac{4\pi}{3}(r^2 + \frac{18}{\pi r}).$
$Given\ that\ the\ volume\ of\ the\ capsule$ is $12mm^3$,
$determine\ the\ minimum\ value\ of\ S$

121. $A\ container\ in\ shape\ of\ an\ inverted\ right\ circular\ cone,$ $the\ height\ is\ 12cm\ and\ in\ a\ vertical\ angle\ 60^0.$
$A\ tap\ delivers\ water\ in\ the\ container$ at a $rate\ which\ is\ proportional\ to\ the\ depth$ $of\ water\ collected\ in\ the\ container$ at $any\ time.$ (i) $show\ that\ the\ rate\ at\ which\ the\ water\ level\ is\ riding$ is $proportional\ to\ the\ depth.$ (ii) $if\ it\ takes\ 1\ minute\ to\ collect\ a\ depth$ of 6cm of $water,\ calculate\ the\ time\ it\ takes\ to\ fill\ the\ whole\ container.$

$Solve\ the\ following\ differential\ equations..$

122. $\frac{dy}{dx} - y = x^3e^{x^2}.$

123. $\frac{dy}{dx} + ytanx = cosx.$

124. $\frac{dy}{dx} = xy^2 - x$ given $y(0) = 2$

125. $(1-x^2)\ \frac{dy}{dx} - xy^2 = 0$, given $y(0) = 1$

126. $x^2\ \frac{dy}{dx} = x^2 + xy + y^2.$

127. $A\ student\ walks\ to\ school\ at\ a\ speed\ proportional\ to$ $the\ square\ root\ of\ the\ distance\ he\ still\ has\ to\ cover.$
$If\ the\ student\ covered$ 900m in 100

minutes and the school is 2500m from home, find how long he takes to get to school

128. *The population of a certain type of fish in a reserved part* of a *lake is allowed to change at rate* $\frac{dx}{dt} = 10 - 2t$, *where X is a population at time t years.* (a)*If the population is 2000 birds initially, show that* $x = 2000 + 10t - t^2$. (b) *find how long the population takes to grow to its maximum population.*

(c)*calculate the population of birds at the instant* when it's *decreasing at* 14*birds per day.*

129. *Given that* $y = \frac{x(x-3)}{(x-1)(x-4)}$. (i) *show that the curve doesn't have* the *turning points.* (ii)*find the equation of asymptotes and hence sketch* the *curve.*

130. *The curve with the equation* $y = \frac{ax+b}{x(x+2)}$ *where a and b are constants has a* turning point *at* $(1, -2)$. *Find values of a and b,* (ii)find *the equation of the asymptotes and hence sketch the curve.*

131. *Sketch the curve* $f(x) = x^2(x + 2)$ and *hence sketch the curve* of $\frac{1}{f(x)}$.

132. *Given that* $y = \frac{9}{3+x^2}$, *find the intercepts, asymptotes and hence sketch the curve.*

NGS SEMINAR QUESTIONS

P425/2: APPLIED MATHEMATICS.

STATISTICS AND PROBABILITY

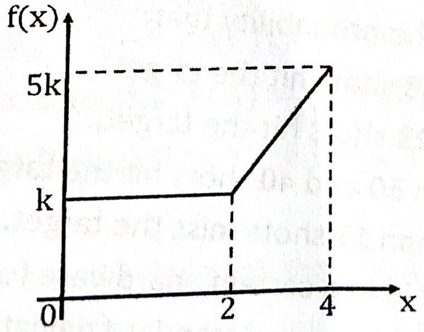
1) A continuous random variable X is uniformly distributed in the interval $[\alpha, \beta]$. The lower quartile is 5 and the upper quartile is 9. Find;
   a) Values of α and β, hence the p.d.f.
   b) E(X) and Var (x).
   c) P (4< X < 8)

2) A random variable y has a Cumulative distribution function, $F(y)$ given below.

$$F(y) = \begin{cases} 0 & : \ y \leq -1 \\ k(y+1) & : \ -1 \leq y \leq 0 \\ k(2y+1) & : 0 \leq y \leq 1 \\ 3k & : y \geq 1 \end{cases}$$

Determine the;
   (i) value of k
   (ii) p. d. f of y.
   (iii) mean, μ.
   (iv) $P(|y - \mu| < \frac{1}{3})$.

3) The p.d.f. of a continuous random variable X is distributed as follows:



Find:
   (i) the value of k
   (ii) the equations of the p.d.f.

(iii) $P(1 < X < 3)$

(iv) the cumulative distribution function $F(x)$ and sketch it.

4) (a) Two events A and B are said to be independent. Show that A' and B' are also independent.

(b) It is assumed that pupils of a certain school hate their teachers due to their constant abuses, over testing and due to over caning with corresponding probabilities of 30%, 50% and 20%. Given the corresponding probabilities that the student would quit the school as 2%, 5% and 3%.

(i) Find the probability that the pupil quits the school.

(ii) Given that the pupil quits the school, find the probability that it was due to over caning

5) Independent observations are taken from a normal distribution with a mean of 30 and a variance of 5. Find the;

(a) probability that the average of 40 observations exceed 30.5.

(b) value of n such that the probability that the average of n observation exceeds 30.5 is less than 1%.

6) A soldier is thrice as likely to hit the target as missing it. If 48 shots are fired, find the probability that:

a) exactly **28** shots hit the target.

b) at least **29** shots hit the target.

c) between **30** and **40** shots hit the target.

d) fewer than **17** shots miss the target.

6. The length of rods sold in a certain hardware follow a normal distribution with a mean of 17.2 meters and standard deviation of 3.6 meters.

a) Find the 90% central limits of the length of the rods.

b) If 25 rods are chosen at random find the probability that the mean length of the rods will lie between 16m and 18m.

c) Find the probability that at least three among the five rods picked at random will have a length of more than 20 meters.

7. Two soldiers A and B in that order take turns shooting a bullet at a target. The first one to hit the target wins the game. If their chances of hitting the target on each occasion they shoot are $\frac{1}{3}$ and $\frac{1}{4}$ respectively, find the chance that:
   a) **A** wins the game on his third shot.
   b) **A** wins the game.
8. A task in mathematics is given to three students whose chances of solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$. Find the probability that:
   a) the task is solved.
   b) only one student solves it.
   c) at least two of them solved it.

9. The p.d.f. of a discrete random variable, X is as follows:

$$P(X = x) = \begin{cases} \beta x \ , & x = 1,2,3 \\ \beta(x+1) \ , & x = 4,5 \\ 0 \ , & \text{otherwise} \end{cases}$$

   Find;
   a) the value of **k**
   b) $P(2 \leq X < 5)$
   c) mode and median of **X**
   d) E(X)
   e) Var(Y) if $Y = 2x - 1$
10. Mutually exclusive events A and B are such that $P(A \cup B) = 0.75$ and P(A) = 0.27, find:
    (i) $P(A' \cup B)$ (ii) $P(A' \cap B')$ (iii) $P(A \cap B)'$
11. Events A and B are such that $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{17}{24}$. Find:
    (i) $\mathbf{P(A \cap B)}$ (ii) $\mathbf{P(A' \cap B)}$ (iii) $\mathbf{P(A' \cap B)}$ (iv) $\mathbf{P(A' \cup B')}$

12. Exhaustive events A and B are such that 5P(A) = 4P(B) and $P(A \cap B) = \frac{1}{5}$ .Find: (i) P(A) (ii) P(A / B)
13. The mark, X, scored by candidates in a test is normally distributed with mean, μ and standard deviation, σ. Given that 80% of the candidates scored above 30 marks and 20% were awarded a distinction with at least 70 marks. Find the;

a) Value of μ and σ.

b) Number of candidates who passed out of the 500 who sat for the test if the pass mark was 50 marks.

14. Three pens are drawn without replacement from a bag containing **5** red and **3** blue pens. Find the:

a) probability distribution for the number of red pens drawn.

b) expected number of red pens drawn.

15. probability of obtaining at least **2** red pens

16. Bag **X** contains **5** red and **4** white beads, bag **Y** contains **7** red and **5** white beads, while bag **Z** contains **3** red and **5** white beads. A bag is selected at random and two beads are picked from it without replacement. Find the probability that:

a) they are of different colours.

b) bag **Y** is selected given that the beads drawn are of the same colour.

17. a) A random variable has a distribution of the form,

$f(x) = c\left(\frac{4}{5}\right)^x$, $x = 0,1,2,, \dots.,$ Find c and P $\left(x \geq 2/_{x \leq 6}\right)$.

b) Okello played 15 chess games. The probability that he wins a game is 0.6.

(i) Find the probability that he won between 6 to 10 games.

(ii) Calculate the most likely number of games he won.

c) Mary and Peter play a game in which they each throw a die in turn until someone throws a six. The person who throws a six wins the game, Peter starts the game. find the probability that she wins.

18. A random sample of 100 observations from a normal distribution with mean, μ gave the following data. $\sum x = 8200$. And $\sum x^2$= 686800. Calculate the;

(i) 99% confidence limits for μ.

(ii) 96 % confidence limits for μ.

19. The table below show the frequency distribution of marks obtained by a group of students in a paper two mathematics examination.

Marks (%)	10 -	20 -	35 -	45 -	65 -	80 -	90 -
Frequency density	1.8	2.4	5.8	3.3	1.2	0.4	0

a) Calculate the

(i) Modal mark

(ii) mean mark,

(iii) standard deviation,

(iv) number of students who scored above 54%.

b) Draw a cumulative frequency curve and use it to estimate the;

i) P_{10}-P_{60} range,

ii) number of students who scored below 40%,

iii) least mark if 20% of the students scored a distinction.

20. The table below shows the consumer prices per unit of commodities A, B, C and D in 2015 and 2017, with corresponding weights.

	Price in Shs.		
Commodity	2015	2017	Weights
A	9000	11000	9
B	6000	7000	6
C	x	y	2
D	6000	8000	3

Given that the simple aggregate index and the cost-of-living index were 124 and 122.5 respectively in 2017 basing on 2015, find the values of x and y. (05 marks)

(b) The table below show the items and quantities bought by a household in 2020 and 2022.

Item	Price 2020=100	Price 2022	Weight
Rice	12500	17500	12
Bread	4000	5500	20
Milk	2000	2500	15
Vegetables	1200	1800	30
Fruits	6000	7200	25

Calculate the

(i) simple aggregate price index

(ii) composite index

If the household spent shs 250,000/= in 2020, how much did it spend in 2022

21. The marks of **8** students in **GP** and **ICT** were as follows:

GP (x)	72	80	50	64	72	56	50	60
ICT (y)	78	79	65	60	85	67	54	65

(a) Plot a scatter diagram for the data. Comment on the relationship between the two tests.

(b) Draw a line of best fit for the scatter diagram and use it to find:

(i) x when y = 70. (ii) y when x = 55.

(c) Calculate the rank correlation coefficient for the scores in the two tests. Comment on your result based on 1% level.

22.The table below shows the weights in kg of 100 babies:

Weights	2.0	2.5	4.5	6.0	7.0	8.0
No of babies	35	20	20	10	5	5

a) Calculate the mean, variance and median for the above data.

Assuming this was a sample taken from a normal population, find the 97.5% confidence interval for the mean weight of all babies

23. A random variable **X** is normally distributed such that P(X < 81) = 0.8849 and P(X > 66) = 0.9641. Find:

a) the mean and standard deviation of **X**.

b) P(69 < X < 83).

c) the interval which contains the middle 95% of distribution.

24. A random sample of 36 items drawn from a normal population is such that the 95% confidence interval for the mean of all the items is [67.9, 77.7] Find the 90% confidence limits for the mean of all the items.

NUMERICAL METHODS

25. The numbers x = 14.8, y = 8.75 and z = 12.4 were calculated with errors of 4%, 3% and 5% respectively. Find the range within which $\frac{y}{x-z}$ lies

26. The numbers x and y were estimated with errors Δx and Δy respectively.

   a) Show that the maximum relative error in the product xy and quotient $\frac{x}{y}$ is the same.

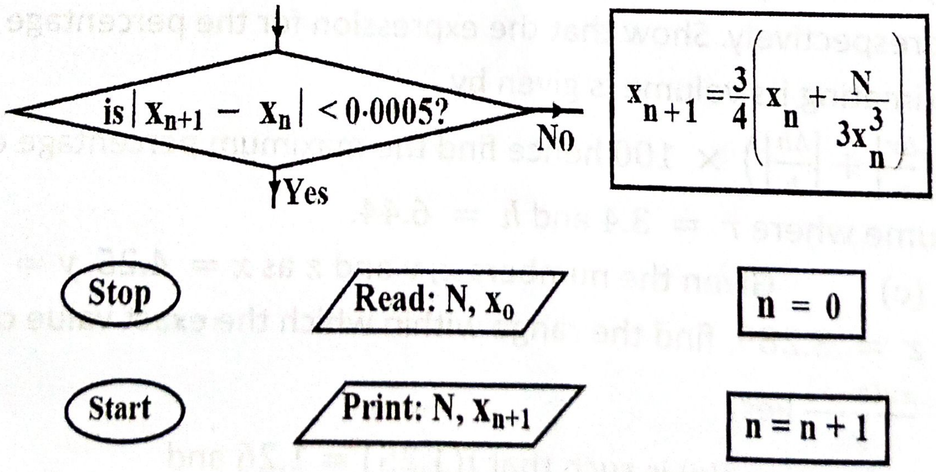
   b) If $x = 4.96$ and $y = 2.243$ were rounded off to the given number of decimal places, calculate the:

      (i) absolute error in $\frac{x}{y}$

      (ii) limits within which $\frac{x}{y}$ is expected to lie, correct to 3 decimal places

27. By drawing graphs of $y = e^{2x}$ and $y = 5x + 1$, on the same axes, show that the equation $e^{2x} - 5x - 1 = 0$ has a root between 0 and 1. Use Newton Raphson Method to determine the root correct to 3 decimal places.

28. The iterative method for solving an equation is described by the following parts of the flowchart:



(i) By rearranging the given parts, draw a flow chart that shows the algorithm for the described method.

(ii) Using N = 38 and x_0 = 2, perform a dry run for the flow chart and state its purpose.

29. Draw a flow chart that computes and prints the product of the first **7** natural numbers. Hence perform a dry run of your flow chart.

30. a) Given the two iterative formulae $x_{n+1} = \frac{2x_n^{\ 3}+1}{3x_n^{\ 2}-5}$ and $x_{n+1} = \sqrt{5+\frac{1}{x_n}}$ using $x_0 = 2.3$, deduce with a reason the more suitable formula, hence give the root to 2dps

b) Use the trapezium rule with six ordinates to estimate $\int_1^2 6^{-2x}\,dx$ correct to three decimal places.

(i) Find the exact value of $\int_1^2 6^{-2x}\,dx$.

(ii) Calculate the percentage error made in your estimation in (a).

31. (a) The volume V of a cone can be calculated from the measurement of radius, r and height, h where Δv, Δr and Δh are errors in volume, radius and height respectively. Show that the expression for the percentage error in approximating its volume is given by

$\left(2\left|\frac{\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right|\right) \times 100$ hence find the maximum percentage error in the volume where $r = 3.4$ and $h = 6.44$.

(c) Given the numbers x, y and z as $x = 4.25, y = 8.9$ and $z = 3.289$, find the range within which the exact value of $\frac{xz(z-x)}{y}$ lies.

32. A certain function f(x) is such that f(1.25) = 1.26 and $f^{-1}(1.19) = 1.35$. Use linear interpolation or extrapolation to find the value of:

(i) $f^{-1}(1.22)$ (ii) $f(1.15)$

33. The bus charges over distances of 4km and 10km from town P are UGX 2800 and UGX 5800 respectively. Estimate the:

(i) charge that is worth 7km from P.

(ii) distance from P worth UGX 5300.

(iii) travel refund when a ticket of 8km is cancelled to 5km.

34. (a) The table below is an extract from the table of $Cos\ x°$

$x = 80°$	0	10'	20'	30'	40'
$Cos\ x$	0. 1736	0.1708	0.1679	0.1650	0.1622

Find the;

(i) $Cos\ 80°\ 36'$

(ii) $Cos^{-1}(0.1685)$

(b) Show that the equation $In\ x + x - 2 = 0$ has a root between $x = 1.5$ and $x = 1.6$. Use linear interpolation thrice to determine the root of the equation correct to 3 dps.

35. (i) Show that the iterative formula for finding the reciprocal of a number A is given by $x_{n+1} = x_n(2 - Ax_n)$.

(ii) Draw a flow chart that reads A and the initial approximation x_0, computes and prints A and its reciprocal after 2 iterations and gives it with an error of less than 0·0001.

(iii) Perform a dry run for your flow chart using A = $7/16$ and $x_0 = 2$.

MECHANICS

36. (a) A mass of 5kg is pulled up at a Constant speed along a rough surface inclined at 20° to the horizontal. The angle of friction is 15° and the force pulling the mass is 30N. Find the angle between the 30N and the line of greatest slope.

(b) A body of mass 30kg is fixed at the lower end of a light inextensible string whose other side is fixed on a ceiling. A horizontal force, P and a vertical force of 2.8 N keep the body in equilibrium with the string inclined at 60 ° to the vertical. Find P and the tension in the string.

37. ABCDEF is a regular hexagon. Forces of 3.5N, 4N, 6N, 1.5N and 2N act along the sides BC, AC, DA, AE and DE respectively with the direction of the forces being indicated by the direction of letters. Find the;
    a) magnitude and direction of the resultant force.
    b) Equation of line of action of the resultant force and hence or otherwise find where it cuts AB.

38. (a) A force $\boldsymbol{F}$ of magnitude 28N acts in the direction $-\boldsymbol{i} + \sqrt{3}\,\boldsymbol{j}$. Show that $\boldsymbol{F} = (-14\boldsymbol{i} + 14\sqrt{3}\boldsymbol{j})$.

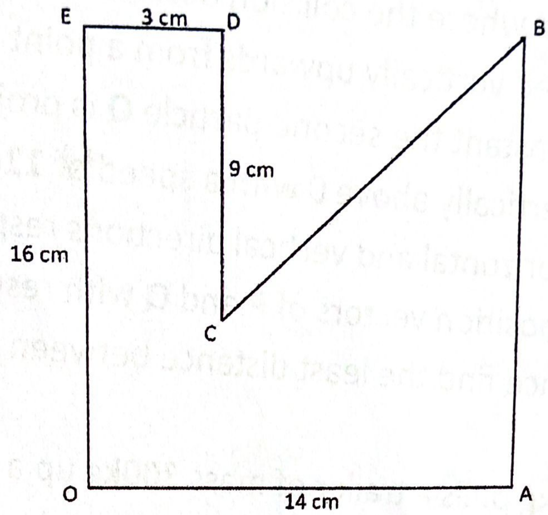
(b) Two forces A and B have magnitudes $\boldsymbol{u}$ and $\boldsymbol{v}$ and act in directions $\boldsymbol{i} - \boldsymbol{j}$ and $\boldsymbol{i} + \boldsymbol{j}$ respectively. Given that the resultant of A and B is equal to $\boldsymbol{F}$ in (a) above, show that $\frac{u+v}{v-u} = \frac{1}{\sqrt{3}}$.

39. A non uniform rod AB of mass 12 kg has its Centre of gravity at a distance of $\frac{3}{4}$ AB. The rod is Smoothly hinged at A. It is maintained in equilibrium at 60° above the horizontal by a light inextensible string fixed at B and at 70° to AB. Calculate the magnitude and direction of the reaction at A.

40. A particle of mass 2 kg is acted upon by a force $\boldsymbol{F} = 54t^3\,\boldsymbol{i} + 24t^3\,\boldsymbol{j} - 18t\,\boldsymbol{k}$ where t is time. Initially, the particle is located at a point with position vector $(1, 0, 0)\,m$ and moving with velocity $(1, 0, 1)ms^{-1}$.
    (a) Determine its;
        (i) distance from the origin after 2s.
        (ii) velocity at t = 1s.

(b) Show that the work done in the time interval t = 1s to t = 2s is equal to the change in the Kinetic energy of the particle.

41. (a) Find the centre of gravity of the lamina shown below.



(b) If the lamina is suspended from 0, find the angle that OE makes with the vertical.

42.a) An aircraft, at a height of 180m above horizontal ground and flying horizontally with a speed of $70ms^{-1}$, releases emergency supplies. If these supplies are to land at a specific point, at what horizontal distance from this point must the aircraft release them?

b) A particle is projected vertically upwards with velocity ums^{-1} and after t seconds another particle is projected vertically upwards from the same point with the same initial velocity. Prove that they will meet at a height of $\frac{4u^2 - g^2t^2}{8g}$

c) A car approaching a town covers two successive half-kilometers in 16 and 20 seconds respectively. Assuming the retardation is uniform. Find the further distance the car runs before stopping.

43.At 4:00 pm, a battle ship is at a place with position vectors $-6i + 12j\ km$ and it is moving with a velocity$16i - 4jkm/h$. At 4:30 pm, a cruiser is at a place with position vector $12i - 15jkm$ and is moving with a velocity $8i +$

$16\boldsymbol{j}km/h$.Show that if these velocities are maintained, the ships will collide. Find when and where the collision occurs.

44. A particle **P** is projected vertically upwards from a point **0** with a speed of **16ms^{-1}**. At the same instant the second particle **Q** is projected horizontally from point **A** .**25m** vertically above 0 with a speed of **12ms^{-1}**. using **i** and **j** as unit vectors in the horizontal and vertical directions respectively. find expressions for the position vectors of **P** and **Q** with respect to 0 at time t after projection, hence find the least distance between the particles (take g =10ms^{-2})

45. A car of mass **1200kg** pulls a trailer of mass **300kg** up a slope of **1 in 100** against a constant resistance of **0.2N per** kg. Given that the car moved at a constant speed of **1.5ms^{-1} for 5 minutes**, calculate the;

(i) tension in the tow bar.

(ii) work done by the car engine during this time

(iii) a car has an engine that can develop **15kw**. If the maximum speed of the car on a level road is **120kmh^{-1}**, calculate the total resistance at this speed.

END

OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2019

STATISTICS

1. (a) A sample consists of 15 numbers, 2,4,7,3,5,6,3,6,10,7,8,9,3,4,3. Find:
   (i) the mean. (ii) standard deviation of the sample. (iii) un biased estimate of population standard deviation

   (b) The following values were recorded in a frequency table; $\sum f(x) = 600$, $\sum fx^2 = 10{,}000$. The standard deviation was calculated as 5. Find the total number of items that were considered.

2. Study the table below;

Marks	20 - 24	25 -34	35 - 38	39 - 44	45 - 49	50 - 54
Frequency	6	10	8	9	5	2

   (a)(i) Construct a histogram using this data (ii) Use the histogram to estimate the mode.
   (b). Calculate the mean mark.

3. The table shows the distribution of heights H among spinach plants in a garden

Heights (H cm)	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70	≤ 80
Number of plants	10	16	21	27	14	12	8	2

   (a) Calculate; (i) the mean (ii) the median height (iii) standard deviation

   (b). Plot an ogive for this data and use it to estimate the interval within which the central 80% of the heights lie.
   (c) Calculate the 10^{th} and 70^{th} percentile range.
   (d) If a sample of 80 plants is taken randomly from the garden, find the 95% confidence limit for the mean height of the total plants.

4. The Mock examination marks and the Average final examination marks of a certain school are given in the following table

Mock Marks	28	34	36	42	52	54	60
Average final examination Marks	54	62	68	70	76	66	74

   (a) (i) Plot the marks on the scatter diagram and comment on the relationship between the two marks.
   (ii) Draw a line of best fit and use it to predict the average final mark of a student whose mock mark is 50.

(b) Calculate the rank correlation coefficient between the marks and comment on your result at 5% level of significance.

5. The table below shows the prices of four commodities and their weights in 2006 and 2007.

Commodity	Price (**U shs**)		Weight
	2006	**2007**	
Banana (1 bunch)	3000	8000	4
Meat(1kg)	2500	3000	3
Milk (1 litre)	300	400	2
Sugar (1 kg)	1500	1800	1

Taking 2006 as the base year, find:

(i) the price relative for each commodity. (ii) Weighted price index for all the commodities

PROBABILITY

6. (a) A and B are two independent events with A twice as likely to occur as B. If $P(A) = \frac{1}{2}$ find:

(i) $P(A \text{ or } B \text{ but not both})$ (ii) $p(\frac{A}{B'})$

(b) At a certain university, the intake policy is such that 40% and 60% of the applicants taken are women and men respectively. The corresponding percentages on government sponsorship are 50% and 40%; the rest are privately sponsored

(i). What is the probability that a student picked at random is a man given that he is on government sponsorship

(ii)Twenty students are picked at random, find the mean number of privately sponsored females in the group.

7. (a) X is a discrete random variable which takes all integers from 1 to 40; such that:
$P(X = x) = kx; \quad x = 1,2,3, \dots ,40.$

(i) Find the value of the constant k; and compute the standard deviation of X.

(ii). Find;$P(X < 35/X > 20)$

(b) A random variable X has a cumulative distribution function given below.

$$F(x) = \begin{cases} 0, & x \leq 0 \\ ax, & 0 \leq x \leq 1 \\ \frac{x+b}{3} & 1 \leq x \leq 2 \\ 1, & x \geq 2 \end{cases}$$

Find; (i) the value of a and b (ii) $P(X < 1.5/X > 1)$ (iii). Obtain and sketch $f(x)$ (iv) Mean of X

8. (a) The weights of ball bearings are normally distributed with mean 25gram and standard deviation 4 grams. If a random sample of 16 ball bearings is taken, find the;

(i) Probability that the mean of the sample is between 24.12 grams and 26.73 grams.

(ii) Interquartile range.

(b) A random sample of 120 girls taken from a normally distributed population of school girls gave a mean age of 16.5 and variance of 18. Determine the 97% confidence interval for the mean age of all the school girls.

9. (a) A box A contains 1 red, 3 green and 1 blue balls. Box B contains 2 red, 1 green and 2 blue balls. A balanced die is thrown and if the throw is a six, box A is chosen, otherwise box B is chosen. A ball is drawn at random from the chosen box. Given that a green ball is drawn, what is the probability that it came from box A?

(b) The normal distribution $X \sim N(\mu, \delta^2)$. Given that $P(X > 80) = 0.0113$ and $P(X < 30) = 0.0287$, find the mean μ and standard deviation δ.

NUMERICAL METHODS

10. Given that $A = |a||b|sin\theta$;

(a) Show that the maximum possible relative error in A is given by $\left|\frac{\Delta a}{a}\right| + \left|\frac{\Delta b}{b}\right| + |\Delta\theta|cos\theta$.
where Δa, Δb and $\Delta\theta$ are small numbers compared to $a, b \ and \ \theta$ respectively.

(b) Write down the possible errors in each measurement where $a = 2.5cm, b = 3.4cm \ and \ \theta = 30^0$.

(c) Value within which the area is enclosed.

(d) Maximum percentage relative error in the area.

11. Show that the equation $\pi \sin x - x = 0$ has a root between $\frac{\pi}{2}$ and π. Hence use linear interpolation only once to find the root correct to three significant figures.

(b) Using the trapezium rule with 7 ordinates, estimate $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2-\cos x}}\,dx$, correct to 3 decimal places.

(c) Calculate the percentage error in your estimation in (a) above.

(d) State how the above error can be reduced.

12. (a) Using the graphical method, show that the equation $3x^3 + x - 4 = 0$ has a root between 0 and 2.

(b) Use linear interpolation once to find the initial approximation of the root.

(c) Hence using Newton Raphson's method, find the root correct to three decimal places.

13. The correct fares a special car hire company charges are proportional to the squares of the distances travelled. The only correct amounts the drivers of a company declare are shs 5,000 and shs 9,800 for distances of 10km and 14 km travelled respectively, and they expect to cheat the travelers by charging the rest of the distances by linear interpolation or extrapolation. Find;

(a) The amount by which one who travels 13km is cheated.

(b) What distance, on the wisdom of the drivers, one travels for free.

14. (a) Show that the Newton Raphson's formula for approximating the natural logarithm of the K^{th} root of a number N is given by $x_{n+1} = \frac{1}{K}(Kx_n - 1 + Ne^{-Kx_n})$.

(b) Draw a flow chart that;

(i) reads the number N, K and the initial approximation x_0.

(ii) Computes and prints the root, correct to 3dps.

(c) Perform a dry run for $K = 4, N = 20\ and\ x_0 = 0.65$.

MECHANICS

15. (a) A particle of mass 5 Kg resting at point (1, -4, 4) is acted upon by three forces. F_1 =3i + 3j, F_2 = 2j + 4k, F_3 = 2i + 6k. Find the position and momentum of the particle after 4 seconds.

(b)A particle of mass 4kg moves with a velocity of $e^t\boldsymbol{i} + 2e^{2t}j - sintk$.Find the power developed after 4 seconds.

16. (a) A body of mass 5 kg slides a distance of 8 m down a rough plane inclined at an angle of $sin^{-1}\left(\frac{4}{5}\right)$ to the horizontal. If the coefficient of friction is 0.4, find the velocity attained by the body.

(b) A particle of mass 50 kg is suspended by two light inelastic strings of lengths 9 m and 12 m attached to two points distant 15 m apart. Calculate the tensions in the strings.

17. A light inextensible string has one end attached to a ceiling. The string passes under a smooth moveable pulley of mass 2 kg and then over a smooth fixed pulley. Particle of mass 5 kg is attached at the free end of the string. The sections of the string not in contact with the pulleys are vertical. If the system is released from rest and moves in a vertical plane, find the:

(a) acceleration of the system. (b) tension in the string.

(c) distance moved by the moveable pulley in 1.5 s.

18. (a) A particle is executing simple harmonic motion with amplitude 2metres and period 12 seconds. Calculate the maximum speed of the particle.

(b) If initially, the particle was moving at a maximum speed, find the;

(i) distance moved by the particle until its speed is half the maximum value

(ii) time taken by the particle to travel this distance.

19. (a) A, B, C, D, is a square. Forces of magnitudes 9N, 5N, and $3\sqrt{2}$ N act along $\overrightarrow{AB}, \overrightarrow{BC}\ and\ \overrightarrow{BD}$ respectively. Find the magnitude and direction of their resultant force.

(b) A non-uniform ladder AB whose centre of gravity is 2m from end A is of length 6m and weight, W. The ladder is inclined at an angle θ to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ. If the ladder is about to slip when a man of weight 5W ascends two-thirds of the way up the ladder, show that $tan\theta = \frac{11-7\mu^2}{18\mu}$

END

OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2024

ORGANISED ON SATURDAY 05TH OCTOBER 2024.

ALGEBRA

1. (a) The sum of n terms of a particular series is given by $S_n = 17n - 3n^2$;
(i) Find an expression for the n^{th} term of the series.
(ii) Show that the series is an Arithmetic progression.

(b) A student deposits shs. 1,200,000 once into her savings account on which an interest of 8% is compounded per annum. After how many years will her balance exceed shs, 200,000?

(c) A piece of land of area $50{,}100m^2$ is divided in such a way that the areas of the plots are in an Arithmetic progression (AP). If the area of the smallest and the largest plots are $2m^2$ and $1000m^2$ respectively, find the;

(i) Number of plots in the piece of land.
(ii) Total area of the first 13 plots to the nearest square metres.

2. (a) Solve the inequality $\frac{x+3}{x-2} \geq \frac{x+1}{x-2}$

(b) Given the curve, $y = \frac{(x-1)(x-4)}{(x-5)}$
(i) Find the range of values of y for which the curve doesnot lie and hence deduce the coordinates of the turning points.
(ii) Show that $y = x$ is an asymptote and state the other asymptote
(iii) Sketch the curve.

3. (a) Solve for x; $64x^{\frac{2}{3}} + x^{\frac{-2}{3}} = 20$
(b) Find the ratio of the coefficient of x^7 to that of x^8 in the expression of $\left(3x + \frac{2}{3}\right)^{17}$
(c)(i) Expand $(1 + x)^{-2}$ in descending powers of x including the term in x^{-4}
(ii) If $x = 9$, find the % error in using the first two terms of the expression in c(i) above.

4. (a) Given that W and Z are two complex numbers, solve the simultaneous equations;
$$3Z + W = 9 + 11i$$
$$iW - z = -8 - 2i$$
(b) Use Demoivre's theorem to simplify; $\frac{[\sqrt{3}(cos\theta + isin\theta)]^8}{[3cos2\theta + 3isin2\theta]^3}$
(c) If $(1 + 3i)z_1 = 5(1 + i)$, show that the locus of $|z - z_1| = |z_1|$ where Z is a complex number is a circle and find its Centre and radius
(d) Given that the factors $(x - 1)$ and $(x + 1)$ are factors of the polynomial, $f(x) = ax^4 + 7x^3 + x^2 + bx - 3$, find the values of the constants a and b. Hence, find the set for real values of x for which $f(x) > 0$

TRIGONOMETRY

5. (a) Prove that $\tan(\theta + 60^0)\tan(\theta - 60^0) = \frac{tan^2\theta - 3}{1 - 3tan^2\theta}$

(b) Show that $-\sqrt{5} \leq cosx + 2sinx \leq \sqrt{5}$

(c) Express $10cosxsinx + 12cos2x$ in the form$Rsin(2x + \beta)$, where R is positive and β is an acute angle. Hence find the maximum and minimum values of $10cosxsinx + 12cos2x$ and state clearly the values of x when they occur for $0^0 \leq x \leq 360^0$.

6. (a) Solve the equation: $\frac{4sin^2\theta}{cosec\theta} + \frac{3}{cosec^2\theta sec\theta} = sin^2\theta$ for $0^0 \leq \theta \leq 360^0$

(b) (i) Prove that $\frac{sin2A+cos2A+1}{sin2A+cos2A-1} = \frac{\tan(45^0+A)}{tanA}$

(ii) Show that $\frac{sin\theta cos2\theta+sin3\theta cos6\theta}{sin\theta sin2\theta+sin3\theta sin6\theta} = cot5\theta$

(c) Show that $\frac{sin\theta}{1-cos\theta} = cot\frac{\theta}{2}$. Hence solve $tan\frac{\theta}{2} = \sqrt{3}sin\theta$ for $0^0 \leq \theta \leq 180^0$

7. (a) Given that X, Y, Z are angles of a triangle. Prove that $\tan\left(\frac{X-Y}{2}\right) = \left(\frac{x-y}{x+y}\right) cot\left(\frac{Z}{2}\right)$, hence solve the triangle if $x = 9cm, y = 5.7cm$ and $z = 57^0$

(b) Prove that $Sin[2sin^{-1}(x) + cos^{-1}(x)] = \sqrt{1-x^2}$

(c) Solve the equation; $2\sin(60^0 - x) = \sqrt{2}cos(135^0 + x) + 1$ for $-180^0 \leq x \leq 180^0$

8. (a) If $tanx = \frac{7}{24}$, and $cosy = \frac{-4}{5}$ where x is reflex and y is obtuse, find without using tables or calculators the value of $sin(x + y)$

(b) In a triangle ABC, $\overline{AB} = 10cm, \overline{BC} = 17cm$ and $\overline{AC} = 21cm$ calculate the angle BAC.

(c) Solve the equation $sin3x + sin7x = sin5x$ for $0^0 \leq x \leq 90^0$

(d) (i) Given that $2A + B = 135$ show that $tanB = \frac{tan^2A-2tanA-1}{1-2tanA-tan^2A}$

(ii) If α is an acute angle and $tan\alpha = \frac{4}{3}$, show that $4sin(\theta + \alpha) + 3cos(\theta + \alpha) = 5cos\theta$. Hence solve for θ the equation $4sin(\theta + \alpha) + 3cos(\theta + \alpha) = \frac{\sqrt{300}}{4}$ for $-180^0 \leq \theta \leq 180^0$

ANALYSIS

9. (a) The point (2,1) lies on the curve $Ax^2 + By^2 = 11$ where A and B are constants.

If the gradient of the curve at the point is 6. Find the values of A and B.

(b) One side of a rectangle is three times the other. If the perimeter increases by 2%. What is the percentage increase in the area?

(c) A rectangular box without a lid is made from a thin cardboard. The sides of the base are $2xcm$ and $3xcm$ and the height of the box is hcm. If the total surface area is $200cm^2$, show that $h = \left(\frac{20}{x} - \frac{3x}{5}\right) cm$. And hence find the dimensions of the box to give maximum volume.

10. (a) If $y = \frac{\cos x}{x^2}$, Prove that; $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (2 + x^2)y = 0$

(b) Given the parametric equations $x = 3 + 4\cos\alpha,\ y = 5 - 8\sin\alpha$. Find $\frac{d^2y}{dx^2}$

(c) A curve is defined by the parametric equations $x = t^2 - t,\ y = 3t + 4$. Find the equation of the tangent to the curve at (2,10)

(d) Using calculus of small changes, Show that $\cos 44.6^0 = \frac{\sqrt{2}}{2}\left(\frac{900+2\pi}{900}\right)$

11. (a). Show that $\int_1^{10} x\log x^2 dx = 2\left(50 - \frac{99}{4\ln 10}\right)$

(b) Express $\frac{x^3+9x^2+28x+28}{(x+3)^2}$ into partial fractions, hence or otherwise show that;
$\int_0^1 \frac{x^3+9x^2+28x+28}{(x+3)^2} dx = \frac{1}{3}\left(10 + \ln\frac{4}{3}\right)$

(c) Find the integrals; (i) $\int \ln\left(\frac{2}{x}\right) dx$ (ii) $\int (x\cos x)^2 dx$ (iii) $\int \frac{x}{\sqrt{1-3x}} dx$

12.(a) The pressure in an engine cylinder is given by; $P = 8000[1 - \sin(2\pi t - 3)]\text{Nm}^{-1}$ At what time does this pressure reach a maximum and what is the maximum pressure.

(b) Calculate the area enclosed by the curve $y = \sin x$ and the line $y = \frac{1}{2}$, from $x = 0$ to $x = \pi$ and the x-axis.

(c) The area bounded by the curves $y^2 = 32x$ and $y = x^3$ is rotated about the x-axis through one revolution. Show that the volume of the solid of the solid formed is $\frac{320\pi}{7}$ cubic units

(d) Using Maclaurin's theorem, expand $(x + 1)\sin^{-1}(x)$ up to the term in x^2

13. (a) Using the substitution $y = uv$, solve the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$

(b) Given that $\frac{dy}{dx} = e^{-2y}$ and $y = 0$, when $x = 5$, find the value of x when $y = 3$

(c) Solve the differential equation $(1 + x)\frac{dy}{dx} = xy + xe^x$ given that $y(0) = 1$

(d) The rate at which a liquid runs from a container is proportional to the square root of the depth of the opening below the surface of the liquid. A cylindrical petrol storage tank is sunk in the ground with its axis vertical. There is a leak in the tank at an unknown depth. The level of the petrol in the tank originally full is found to drop by 20cm in 1 hour and by 19cm in the next hour. Find the depth at which the leak is located.

VECTORS

14. (a) Point B is the foot of a perpendicular from point $A\ (3,\ 0, -2)$ to the line $\boldsymbol{r}$ where $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

(i) Find the values of λ corresponding to the point B. hence state the coordinates of B.

(ii) Calculate the distance of the point A from the line $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and write down the vector parametric equation of the plane containing point A and the line $\boldsymbol{r}$

(b) Find the area of a parallelogram of which the given vectors are adjacent sides, $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{j} + \boldsymbol{k}$ respectively.

(c) A and B are points (3,1,1) and (5,2,3) respectively and C is a point on the line $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. If angle$BAC = 90^0$, find the coordinates of C.

15. (a) Find the coordinates of the point where the line $\frac{x-3}{5} = \frac{3-y}{-2} = \frac{z-4}{3}$ meets the plane $2x - 3y + 7z - 10 = 0$

(b) The vector $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ is perpendicular to the plane containing the line; $\frac{x-3}{-2} = \frac{y+1}{a} = \frac{z-2}{1}$, find the;

(i) Value of a
(ii) Cartesian equation of the plane

(c) Find the perpendicular distance from the point M (4,-3,10) to the line with vector equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

16. (a) Two planes L_1 and L_2 are defined by $3x - 4y + 2z - 5 = 0$ and $\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ respectively.

Find;
(i) Cartesian equation of plane L_2
(ii) Acute angle between the two planes
(iii) Vector equation of the line of intersection of L_1 and L_2
(b) Given the points L (2,-1, 0), M (4, 7, 6) and N (8, 5,-4). Find the vector equation of the line which joins the midpoint of LM and MN.
(c) Determine the equation of the plane equidistant from the points A (1, 3, 5) and B (2,-4, 4)

17. (a) Find the equation of the line through point A(1,-2,3) perpendicular to the line $\frac{x-5}{2} = \frac{y-2}{1} = \frac{z-1}{3}$
(b) Prove that Points A (-2,0,6) and B(3,-4,5) lie on opposite sides of the plane $2x - y + 3z = 21$
(c) Find the equation of a plane containing points A (1, 1, 1), B (1, 0, 1) and C (3, 2,-1)
(d) Show that the vectors $2i - j + k, i - 3j - 5k$ and $3i - 4j - 4k$ are coplanar
(e) Point R with position vector **r** divides the line segment AB internally in the ratio $\lambda{:}\,\mu$, Show that $r = \frac{a\mu + b\lambda}{\lambda + \mu}$ where a and b are position vectors of A and B respectively. Hence find the position vector of point R which divides AB in the ratio 1:2, given that the position vector of A is $\begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$ and that of B is $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

COORDINATE GEOMETRY

18. (a) A line L passes through the point of intersection of the lines $x - 3y - 4 = 0$ and $y + 3x - 2 = 0$. If L is perpendicular to the line $4y + 3x = 0$, determine the equation of the line L.

(b) Variable point $P(x, y)$ moves such that its distance from point $A(3,0)$ is equal to its distance from the line $x + 3 = 0$. Describe the locus of point P.

(c) Calculate the perpendicular distance between the parallel lines $3x + 4y + 10 = 0$ and $3x + 4y - 15 = 0$

(d) Calculate the area of the triangle which has sides given by the equations $2y - x = 1, y + 2x = 8$ and $4y + 3x = 7$

19. (a) The triangle ABC with vertices A(1,-2), B(7,6) and C(9,2), find:

(i) The equations of the perpendicular bisectors of AB and BC.

(ii) The coordinates of the point of intersection of the perpendicular bisectors

(iii) Find the equation of the circle passing through the three points A,B,C of the triangle above.

(b) Show that the circles $x^2 + y^2 - 2ax + c^2 = 0$ and $x^2 + y^2 - 2by - c^2 = 0$ are orthogonal.

(c) Find the length of the tangent to the circle $x^2 + y^2 - 4x + 9 = 0$ from the point (5,7)

20. (a) Determine the vertex, focus, directrix and axis of the parabola $y^2 - 2y - 8x - 17 = 0$ hence sketch the parabola.

(b) The tangents to the parabola $y^2 = 4ax$ at points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ meet at point T, find the coordinates of T.

(c) If $\left(\frac{1}{2}, 2\right)$ is one extremity of a focal chord of the parabola $y^2 = 8x$, find the coordinates of the other extremity.

(d) If $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$, show that $m = \frac{a}{c}$

21. (a) Show that the parametric equations $x = 1 + 4\cos\theta$ and $y = 2 + 3\sin\theta$ represent an ellipse. Hence determine the coordinates of the centre and the lengths of the semi axes

(b) The normal at the point $P(5\cos\theta, 4\sin\theta)$ on an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ meets the x and y-axes at A and B respectively. Find the mid-point of the line AB

(c)(i) Find the equation of the tangent to the hyperbola whose points are of the parametric form$\left(2t, \frac{2}{t}\right)$.

(ii) Find the equations of the tangents in (i) which are parallel to $y + 4x = 0$

(iii) Determine the distance between the tangents in c(ii).

END

ALGEBRA

1. Express $x^2 - 6x + 5$ in the form $p + (x - r)^2$ and hence deduce the coordinates of the turning points of $\dfrac{32}{x^2 - 6x + 5}$ and state the equation of the line of symmetry.

2. If the sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, show that $ab^2 + bc^2 = 2a^2c$.

3. Given that $(x+1)^2$ is a factor of the polynomial $2x^4 + 7x^3 + px^2 + qx + r$ and has a remainder of 14 when divided by $(x-1)$, find the values of $p,\ q,\ r$.

4. Given a G.P $a + b + \ldots + l$ prove that its sum is $S_n = \dfrac{bl - a^2}{b - a}$.

5. Given that the equations $x^2 + px + q = 0$ and $x^2 + mx + k = 0$ have a common root, show that $(q - k)^2 = (m - p)(pk - mq)$.

6. Given that one root of the equation $x^2 + px + q = 0$ is twice the other, show that $2p^2 = 9q$, hence, find the values of k, if the equation $x^2 - 2(k+2)x + (k^2 + 3k + 2) = 0$, has one root twice the other.

7. If $\log_e a^3b = u$ and $\log_e ab^2 = v$, find a in terms of e, u and v.

8. Solve the equation: $\log_{10} e.In(x^2 + 1) - 2\log_{10} e.Inx = \log_{10} 5$

9. Prove by mathematical induction that for all positive integral values of n, $10^n + 3(4^{n+2}) + 5$ is a multiple of 9.

10i) Prove that the curve $y = \dfrac{4x^2 - 10x + 7}{(x-1)(x-2)}$ cannot lie in the region $-2\sqrt{3}$ and $2\sqrt{3}$.

ii) Solve the inequality: $\dfrac{x+1}{2x-3} \le \dfrac{1}{x-3}$

11. The sum of the first n terms of an A.P is $n^2 + 5n$. Find the first three terms of the series.

12. The first term of a geometric progression is A and the sum of the first three terms is $\frac{7}{4}A$.

    i) Show that there are two possible progressions.

    ii) Given that $A = 4$ find the next two terms of each progression.

13. Expand $\frac{1}{\sqrt{1+x}}$ up to the term in x^2 and by letting $x = \frac{1}{4}$, show that $\sqrt{5} \approx \frac{256}{115}$.

14. Express $\left(-1-\sqrt{3}i\right)^6$ in the form $x + iy$.

15. Describe the locus represented by $2|z-2i| = 3|z+1|$ and hence state the centre and radius.

16. If $\frac{(z-i)}{(z-1)}$ is purely imaginary, show that the locus of z is a circle. State the centre and radius.

17. Given $z = 1 + \cos 4\theta + i\sin 4\theta$, find the modulus and argument of z.

18. Solve for n given that ${}^{n}C_4 = 5 \times {}^{n-2}C_3$.

19. Find the square root of $5 + 12i$.

20. If x is sufficiently small the allow any terms in x^5 or higher powers of x to be neglected, show that $(1+x)^6\left(1-2x^3\right)^{10} \cong 1 + 6x + 15x^2 - 105x^4$.

ANALYSIS

21. Evaluate: $\int_{-1}^{-\frac{1}{2}} \frac{(4x+2)}{(x-1)^4(x+2)^4}\,dx$

22. Use small changes to show that $(16.02)^{\frac{1}{4}} \approx 2\frac{1}{1600}$

23. Use $t = \tan\frac{1}{2}x$, to find the value of $\int_0^{\frac{\pi}{2}} \frac{dx}{3+5\cos x}$

24. Sketch the curve $y=\frac{4+3x-x^2}{x-8}$, clearly find the nature of the turning points and state their asymptotes.

25. An open cylindrical container is made from a $12cm^2$ metal sheet. Show that the maximum volume of the container is $\frac{8}{\sqrt{\pi}}cm^3$.

26. The area enclosed between the parts of the curves $x^2+y^2=1$ and $4x^2+y^2=4$ for which y is positive is rotated about the $x-$ axis, find the volume of the solid generated.

27. Evaluate : $\int_1^2 (x-1)^2 \ In\ x\ dx$

28. Find Maclaurin's expansion of $y=In\frac{(2-x)^2}{(1+x)^2}$, showing the first three non zero terms, hence, find the approximate value of $2In\frac{1.99}{1.01}$ correct to 3 s.f.

29. Integrate $\int\frac{4x^2+x+1}{(x^3-1)}dx$

30. Find the area enclosed by the curves $y^2=4x$ and $x^2=4y$.

31. Evaluate: $\int_0^{1/4}\cos^{-1}2x\,dx$

32. Find the equation of the tangent at the point $(1,\ -1)$ to the curve $y=2-4x^2+x^3$. What are the coordinates of the point where the tangent meets the curve again? Find the equation of the tangent at this point.

33. If $y=\tan\left(2\tan^{-1}\frac{x}{2}\right)$, show that $\frac{dy}{dx}=\frac{4(1+y^2)}{4+x^2}$

34. Differentiate from first principles: $y=\sqrt{\cos x}$ from first principles.

35. A particle is moving in a straight line such that its distance from a fixed point O , $t\,s$ after motion begins is $s=\cos t+\cos 2t\ \ m$, find:

i) the time when the particle first passes through O.

ii) the velocity of the particle at this instant.

iii) the acceleration when the velocity is zero.

36. Given $\frac{dy}{dx} = 2\cos x\sqrt{y+3}$, find y in terms of x if $y\left(\frac{\pi}{2}\right) = 1$.

37. Solve the d.e $2y(x+1)\frac{dy}{dx} = 4 + y^2$, given that $y = 2$ when $x = 3$ express y in terms of x.

38. The price p of a commodity varies in such a way that the rate of change of price with respect to time t hours is proportional to the shortage $D - S$, where $D = 8 - 2p$ and $S = 2 + p$. If the price at $t = 0$ is $\$5$ and after $t = 2$ hours the price is $\$3$. Find the price of the commodity at any time and determine the price of the commodity as time tends to infinity.

39. Show that $\int_1^2 \frac{2x^3 - 1}{x^2(2x-1)}dx = \frac{3}{2} + \frac{1}{2}In\left(\frac{16}{27}\right)$

TRIGONOMETRY

40. Prove that $(\sin 2\alpha - \sin 2\beta)\tan(\alpha + \beta) = 2(\sin^2\alpha - \sin^2\beta)$.

41. Given $\sin(x+\alpha) = 2\cos(x-\alpha)$, prove that $\tan x = \frac{2 - \tan\alpha}{1 - 2\tan\alpha}$

42. Solve: $\cos(2\theta + 45^o) - \cos(2\theta - 45^o) = 1$, for $0^o \leq \theta \leq 360^o$.

43. Prove the identity: $\cos^6 x + \sin^6 x = 1 - \frac{3}{4}\sin^2 2x$.

44. The roots of the equation $ax^2 + bx + c = 0$ are $\tan\alpha$ and $\tan\beta$. Express $\text{sec}(\alpha + \beta)$ in terms of a, b, c.

45. If $a = x\cos\theta + y\sin\theta$ and $b = x\sin\theta - y\cos\theta$, prove that $\tan\theta = \frac{bx + ay}{ax - by}$.

46. Solve: $3\tan^3 x - 3\tan^2 x = \tan x - 1$ for $0^o \leq x \leq \pi$.

47. Prove that $\cos^5 x = \dfrac{\cos 5x + 5\cos 3x + 10\cos x}{16}$.

48. Find the values of p and θ, given that $\cos(p+2)\theta + \cos p\theta = \cos\theta$ for $0^o \le \theta \le 360^o$.

49. Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x+\alpha)$, hence or otherwise solve $10\sin x\cos x + 12\cos 2x + 7 = 0$ in the range $0^o \le x \le 360^o$.

50. Prove that: $\cos^2 2A + \cos^2 2B + \cos^2 2C - 1 = 2\cos 2A\cos 2B\cos 2C$.

GEOMETRY

51. The equation of a circle is given by $x^2 + y^2 + Ax + By + C = 0$, where A, B, C are constants, given that $8A = 6B,\ 6A = 4C$ and $C = 18$, find the coordinates of the centre and the radius.

52. Prove that the line $2x - 3y + 26 = 0$ is a tangent to the circle $x^2 + y^2 - 4x + 6y - 104 = 0$ and hence find the coordinates of the point of intersection.

53. Prove that the circles $x^2 + y^2 - 6x - 12y + 40 = 0$ and $x^2 + y^2 - 4y = 16$ are orthogonal.

54. A point P on the curve is given parametrically by $x = 3 - \cos\theta$ and $y = 2 + \sec\theta$.

Find the: (i) equation of the normal to the curve at the point $\theta = \dfrac{\pi}{3}$

(ii) Cartesian equation of the curve.

(b) Point $P(ap^2,\ 2ap)$ and $Q(aq^2,\ 2aq)$ lie on the parabola $y^2 = 4ax$. Find the locus of the midpoint of the chord PQ for which $pq = 2a$.

55. Find the equation of the normal to the curve $y^2 = 4bx$ at the point $P(bp^2,\ 2bp)$. Given that the normal meets the curve again at $Q(bq^2,\ 2bq)$, prove that $p^2 + pq + 2 = 0$.

56. Show that the curve $x = 5 - 6y + y^2$ represents a parabola and find the directrix and sketch.

57. The normal to the parabola $y^2 = 4ax$ at the point $P(at^2,\ 2at)$ meets the axis of the parabola at G and GP is produced, beyond P to Q so that $\overline{GP} = \overline{PQ}$. Show that the equation of locus of Q is $y^2 = 16a(x + 2a)$.

58. The points $P\left(5p,\ \dfrac{5}{p}\right)$ and $Q\left(5q,\ \dfrac{5}{q}\right)$ lie on the rectangular hyperbola $xy = 25$.

i) Find the equation of the tangent at P and hence deduce the equation of the tangent at Q.

ii) The tangents at P and Q meet at point N, show that the coordinates of N are $\left(\dfrac{10pq}{p+q},\ \dfrac{10}{p+q}\right)$.

VECTORS

59i) Show that the point with position vector $\mathbf{i} - 9\mathbf{j} + \mathbf{k}$ lies on the line $\mathbf{r} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 6\mathbf{j} - \mathbf{k})$.

ii) Show that the line $\dfrac{x-2}{2} = \dfrac{2-y}{1} = \dfrac{3-z}{-3}$ is parallel to the plane $4x - y - 4z = 0$.

iii) Find the shortest distance from the point with position vector $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$ to the line $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

60i) A line passes through the mid point of $A(4,\ 3,\ 2)$ and $B(6,\ 2,\ 1)$. This line is parallel to the plane $6x + 3y + 9z = 11$. Find the equation of the line.

ii) A line passes through the point $P(1,\ -3, -4)$ and is perpendicular to the plane $-4x + 3y + 6z = 10$. If this line meets another plane $6x + 8y + 4z = 11$, find the point of intersection.

61. Two lines have vector equations $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = (4 - \beta)\mathbf{i} + (\beta + 4)\mathbf{j} + (1 + 2\beta)\mathbf{k}$. Find the position vector of the point of intersection of the two lines and the Cartesian equation of the plane containing the two lines.

62. A plane, P_1 passing through the point $A(1,\ 1,\ -3)$ is perpendicular to the line $\frac{x}{2}=\frac{1-y}{3}=\frac{z-3}{2}$. Another plane, P_2 contains the points with position vectors $\boldsymbol{p}=\boldsymbol{i}-2\boldsymbol{j}+5\boldsymbol{k},\ \boldsymbol{q}=-2\boldsymbol{i}-4\boldsymbol{j}+3\boldsymbol{k}$ and $\boldsymbol{r}=7\boldsymbol{i}+3\boldsymbol{j}-8\boldsymbol{k}$. Find

(i) The Cartesian equations of the two planes.
(ii) The angle between the two planes.
(iii) The line of intersection of the two planes.

63. The position vectors of points A and B are **a** and **b** respectively. The point C is such that $\boldsymbol{BC}=\boldsymbol{a}$. Another point D on BC produced is such that AC intersects **OD** at T and $\boldsymbol{OT}:\boldsymbol{OD}=2:3$ with respect to the origin O.

a) Find in terms of **a** and **b**, the position vectors of (i) **BT** (ii) **TD.**

b) If $\boldsymbol{a}=15\boldsymbol{i}+\boldsymbol{12j}+18\boldsymbol{k}$ and $\boldsymbol{b}=-10\boldsymbol{i}+16\boldsymbol{j}+6\boldsymbol{k}$ find to the nearest degree, the angle between **BT** and **TD.** Hence, find the area of triangle BTD.

64a) Find the equation of the plane that is perpendicular to line AB, and contains the point P , such that $AP:PB=2:1$, given that $A(1,0,-5)$ and $B(7,6,7)$.

b) The L_1 passes through the point $P(2,0,-4)$ and is parallel to the line $r=\begin{pmatrix}4\\-3\\1\end{pmatrix}+\mu\begin{pmatrix}2\\1\\2\end{pmatrix}$, Determine

i) the position vector of F, the point of intersection of the plane in (a) above and the line L_1

ii) The shortest distance between $P(2,0,-4)$ and the plane in (a).

iii) $\tan\theta$ where θ is the angle between the line L_1 and the plane in (a).

65a) Find the equation of the plane containing the lines $\frac{x-2}{2}=\frac{y+3}{3}=\frac{z-1}{5}$ and $r=\begin{pmatrix}5\\1\\3\end{pmatrix}+\mu\begin{pmatrix}8\\12\\20\end{pmatrix}$.

b) Find the point of intersection of the line $\frac{x-2}{2}=\frac{y+3}{3}=\frac{z-1}{5}$ and the line given by the equation $\boldsymbol{r}=(4+2t)\boldsymbol{i}+(8-5t)\boldsymbol{j}+(7+4t)\boldsymbol{k}.$

ii) Determine the angle between the two lines in (b) above. **END**

JULY 9th 2011 PAPER 425/1

Algebra

1. Solve for x : $\dfrac{(1-x)^2}{2-x^2} = \dfrac{(1-a)^2}{2-a^2}$

$$\frac{1-2x+x^2}{2-x^2} = \frac{1-2a+a^2}{2-a^2}$$

$$(1-2x+x^2)(2-a^2) = (1-2a+a^2)(2-x^2)$$

$$2-a^2-4x+2a^2x+2x^2-a^2x^2 = 2-4a+2a^2-x^2+2ax^2-a^2x^2$$

$4(x-a)-3(x^2-a^2)+2ax(x-a)=0$ factorise by grouping

$$(x-a)[4-3(x+a)+2ax]=0$$

Either $x=a$ Or $4-3x-3a+2ax=0$

So, $x = \dfrac{3a-4}{2a-3}$

2. Given that the first three terms in the expansion in ascending powers of x of $(1-8x)^{1/4}$ are the same as the first three terms in the expansion of $\left(\dfrac{1+ax}{1+bx}\right)$, find the values of a and b. Hence, find an approximation to $(0.6)^{1/4}$ in the form p/q.

$$(1-8x)^{1/4} = 1+\frac{1}{4}(-8x)+\frac{1}{2!}.\frac{1}{4}.\frac{-3}{4}(-8x)^2+\ldots$$

$$= 1-2x-6x^2-\ldots$$

$$\frac{1+ax}{1+bx} = (1+ax)\{1-bx+b^2x^2+..\}$$

$= \ 1+(a-b)x+(b^2-ab)x^2+...$

Since the first three terms of the expansion are the same.

$a-b=-2, b^2-ab=-6$

Hence b = -3 and a = -5

Thus; $(1-8x)^{1/4} \approx \dfrac{1-5x}{1-3x}$

Substituting $x=0.05,$ we have

$$(1-0.4)^{1/4} \approx \frac{1-0.25}{1-0.15} = \frac{0.75}{0.85} \text{ hence } (0.6)^{1/4} \approx 15/17$$

4. Given $x \rangle 2$ and $\left(\dfrac{x+2}{x}\right)^{-1/2} = a+\dfrac{b}{x}+\dfrac{c}{x^2}+\dfrac{d}{x^3}+-+-$, use the binomial expansion and determine the values of a, b, c, d. By taking $x=100$, find the approximate value of $\left(\dfrac{450}{51}\right)^{1/2}$ correct to 4 decimal places.

$$\left(1+\frac{2}{x}\right)^{-1/2} = 1+-\frac{1}{2}\left(\frac{2}{x}\right)+\frac{-\frac{1}{2}.-\frac{3}{2}}{2!}\left(\frac{2}{x}\right)^2+\frac{-\frac{1}{2}.-\frac{3}{2}.-\frac{5}{2}}{3!}\left(\frac{2}{x}\right)^3+\ldots$$

$$=1-\frac{1}{x}+\frac{3}{2x^2}-\frac{5}{2x^3}+-.., \text{ hence, } a=1,\ b=-1,\ c=\tfrac{3}{2},\ d=-\tfrac{5}{2}$$

When $x=100$

$$\left(\frac{100+2}{100}\right)^{-1/2} = \left(\frac{102}{100}\right)^{1/2} = \left(\frac{51}{50}\right)^{1/2}$$

So, to estimate $\left(\dfrac{450}{51}\right)^{1/2} = \left(\dfrac{9\times 50}{51}\right)^{1/2} = 3\left(\dfrac{50}{51}\right)^{1/2} = \dfrac{1}{3}\left(\dfrac{51}{50}\right)^{-1/2}$

Thus, $\dfrac{1}{3}\left(\dfrac{51}{50}\right)^{-1/2} = 1-\dfrac{1}{100}+\dfrac{3}{20000}-\dfrac{5}{2000000}$

$$\therefore \left(\frac{51}{50}\right)^{-\frac{1}{2}} = 3(1 - \frac{1}{100} + \frac{3}{20000} - \frac{5}{2000000})$$

$$= \frac{1980295}{2000000} x\, 3 = 2.9704425 \approx 2.9704 \text{ correct to 4 d.p.}$$

5. Prove by induction that $8^n - 7n + 6$ is divisible by 7 for all $n \geq 1$.

Let $a_n = 8^n - 7n + 6$

For $n = 1, \quad a_1 = 8 - 7 + 6 = 7$, divisible by 7.

For $n = 2, \quad a_1 = 64 - 14 + 6 = 56$, divisible by 7.

For $n = k, \quad a_k = 8^k - 7k + 6$,

When $n = k + 1, \quad a_{k+1} = 8^{k+1} - 7(k + 1) + 6$

Thus $a_{k+1} - a_k = 8^{k+1} - 7(k+1) + 6 - \left(8^k - 7k + 6\right)$

$$= 8^k.8 - 8^k - 7k + 7k - 7 + 6 - 6$$

$= 7.8^k - 7 = 7\left(8^k - 1\right)$ which is still divisible by 8. Thus if its true for $n = 1, 2, \ldots, k, k+1$, then $8^n - 7n + 6$ is divisible by 7 for all $n \geq 1$.

6. Find the values of λ for which $10x^2 + 4x + 1 = 2\lambda x(2 - x)$ has equal roots.

$10x^2 + 4x + 1 = 4\lambda x - 2\lambda x^2 \quad \Leftrightarrow \quad (10 + 2\lambda)x^2 + (4 - 4\lambda)x + 1 = 0$

For equal roots: $b^2 - 4ac = 0$, so $(4 - 4\lambda)^2 - 4(10 + 2\lambda) = 0$

$16\lambda^2 - 40\lambda - 24 = 0 \quad \Leftrightarrow \quad 2\lambda^2 - 5\lambda - 3 = 0$

$(\lambda - 3)(2\lambda + 1) = 0 \quad \therefore \quad \lambda = 3, \lambda = -\frac{1}{2}$

ALT; Let the roots be α & α

$2\alpha = \dfrac{-(4 - 4\lambda)}{10 + 2\lambda}$ where $\alpha = \dfrac{\lambda - 1}{\lambda + 5}$ and $\alpha^2 = \dfrac{1}{10 + 2\lambda}$

So, $\left(\frac{\lambda-1}{\lambda+5}\right)^2 = \frac{1}{2(\lambda+5)}$, to get $2\lambda^2 - 5\lambda - 3 = 0$

$\therefore \quad \lambda = 3, \lambda = -\frac{1}{2}$

7. Given that $z+2i$ is a factor of $z^4 + 2z^3 + 7z^2 + 8z + 12$, solve the equation $z^4 + 2z^3 + 7z^2 + 8z + 12 = 0$.

If $z+2i$ is a factor, then, $z-2i$, thus the quadratic factor is z^2+4

By long division,
$$\begin{array}{r} z^2 + 2z + 3 \\ z^2+4\overline{\left)z^4 + 2z^3 + 7z^2 + 8z + 12\right.} \\ z^4 + \quad 4z^2 \end{array}$$

Thus, $(z^4+4)(z^2+2z+3) = 0$

$z = \pm 2i, \quad z = \frac{-2 \pm \sqrt{-8}}{2}$

The roots are $2i,\ -2i,\ -1+i\sqrt{2},\ -1-i\sqrt{2}$

NOTE: Candidate should differentiate between **a root** and **a factor**.

8. If $z = \cos\theta + i\sin\theta$, solve the equation $z^{4/3} = i$.

To solve $z^{4/3} = i$, we have $z^4 = i^3 = -i$

$z^4 = \cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi$

$= \cos -\frac{1}{2}\pi + i\sin -\frac{1}{2}\pi$

$= \cos -\frac{1}{2}\pi - i\sin\frac{1}{2}\pi$

$z = (\cos -\frac{1}{2}\pi + i\sin -\frac{1}{2}\pi)^{1/4}$

By De moivres thorem,

$$z = \cos\left(\frac{\frac{\pi}{2} + 2k\pi}{4}\right) - i\sin\left(\frac{\frac{\pi}{2} + 2k\pi}{4}\right) \text{ for } k = 0, 1, 2, 3$$

When $k = 0, \quad z_1 = \cos\frac{\pi}{8} - i\sin\frac{\pi}{8} = 0.9239 - 0.3827i$

$k = 1, \quad z_2 = \cos\frac{5\pi}{8} - i\sin\frac{5\pi}{8} = -0.3827 - 0.9239i$

$k = 2, \quad z_3 = \cos\frac{9\pi}{8} - i\sin\frac{9\pi}{8} = -0.9239 + 0.3827i$

$k = 3, \quad z_4 = \cos\frac{13\pi}{8} - i\sin\frac{13\pi}{8} = 0.3827 + 0.9239i$

ANALYSIS

1. Evaluate: $\int_0^{\frac{a}{2}} x^2\sqrt{(a^2 - x^2)}\,dx$

$$\int_0^{\frac{a}{2}} x^2\sqrt{(a^2 - x^2)}\,dx = \int_0^{\frac{a}{2}} x^2\sqrt{a^2\left(1 - \frac{x^2}{a^2}\right)}\,dx$$

Let $\frac{x}{a} = \sin\theta : dx = a\sin\theta\, d\theta$

$x = a\sin\theta$

$$= \int_0^{\frac{a}{2}} a.(a\sin\theta)^2\sqrt{(1 - \sin^2\theta)}.\, a\sin\theta\, d\theta$$

$x = 0, \quad \theta = 0$

$x = \frac{a}{2}, \theta = \frac{\pi}{6}$

$$= a^4\int_0^{\frac{\pi}{6}} \sin^2\theta\cos^2\theta\, d\theta$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$= \frac{a^4}{4}\int_0^{\frac{\pi}{6}} \sin^2 2\theta\, d\theta$$

$\sin^2 2\theta = \frac{1}{2}(1 - \cos 4\theta)$

$$= \frac{a^4}{8}\int_0^{\frac{\pi}{6}} (1 - \cos 4\theta)\, d\theta$$

$$= \frac{a^4}{8}\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{6}}$$

$$= \frac{a^4}{8}\left[\left(\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3}\right) - 0\right]$$

$$= \frac{a^4}{8}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right)$$

$$= \frac{a^4}{16}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

2. Using the substitution

i) $y = e^x + 1$, find $\int \frac{dx}{(e^x + 1)^2}$

$\int \frac{dx}{(e^x + 1)^2}$ $\qquad dy = e^x\ dx$

$\int \frac{dy}{y^2(y-1)}$, so let $\frac{1}{y^2(y-1)} \equiv \frac{A}{y} + \frac{B}{y^2} + \frac{C}{(y-1)}$

Thus $1 \equiv Ay(y-1) + B(y-1) + Cy^2$

Solving: $A = -1,\ B = -1,\ C = 1$

$$\int \frac{dy}{y^2(y-1)} = \int \frac{-1}{y} dy + \int \frac{-1}{y^2} dy + \int \frac{1}{(y-1)} dy$$

$$= -Iny + \frac{1}{y} + In(y-1) + c$$

$$= In\frac{y-1}{y} + \frac{1}{y} + c$$

$$= In\frac{e^x}{e^x + 1} + \frac{1}{e^x + 1} + c$$

ii) $t = \tan\frac{1}{2}x$, find the value of $\int_0^{\frac{\pi}{2}} \frac{dx}{3 + 5\cos x}$

$\int_0^{\frac{\pi}{2}} \frac{dx}{3 + 5\cos x}$ $\qquad \cos x = \frac{1-t^2}{1+t^2}$, $t = \tan\frac{1}{2}x,\ dt = \frac{1}{2}\sec^2\frac{1}{2}xdx$

$$\int_0^1 \frac{(1+t^2)}{3+5\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2} \qquad x=\tfrac{\pi}{2},\ t=1 \quad x=0,\ t=0$$

$$\int_0^1 \frac{1}{4-t^2}\,dt \qquad \text{let } \frac{1}{4-t^2}\equiv\frac{A}{2+t}+\frac{B}{2-t}$$

$$\frac{1}{4}\int_0^1\left(\frac{1}{2+t}+\frac{1}{2-t}\right)dt$$

$$\frac{1}{4}\left[In\frac{2+t}{2-t}\right]_0^1 = \frac{1}{4}[In3-In1]=\frac{1}{4}In3$$

iii) $t=\tan x$ find $\int\frac{dx}{\cos^2 x+4\sin^2 x}$

$$\int\frac{dx}{\cos^2 x+4\sin^2 x} = \int\frac{\sec^2 xdx}{1+4\tan^2 x} \qquad t=\tan x,\ dt=\sec^2 xdx$$

$$= \int\frac{\sec^2 x}{1+4t^2}\cdot\frac{dt}{\sec^2 x}$$

$$= \int\frac{dt}{1+4t^2}$$

$$= \frac{1}{2}\tan^{-1}2t+c$$

$$= \frac{1}{2}\tan^{-1}2(\tan x)+c$$

iv) $t=e^x$ to find $\int\frac{e^x}{e^x+e^{-x}}dx$

3. Find the equation of a curve whose gradient is given by xe^{x-y} and passes through the point $(0, In2)$.

$\frac{dy}{dx} = xe^{(x-y)}$, separating the variables: i.e $\frac{dy}{dx} = \frac{xe^x}{e^y}$

$\int e^y dy = \int xe^x dx$ let $u = x,\ \frac{dv}{dx} = e^x$, $\frac{du}{dx} = 1,\ v = e^x$

$\therefore e^y = xe^x - \int e^x\ dx$

$\therefore e^y = xe^x - e^x + C$, $x = 0,\ y = In2$

Thus; $e^{In2} = 0e^o - e^o + C, \quad C = 3$

$\therefore e^y = xe^x - e^x + 3$, thus $y = In(xe^x - e^x + 3)$

6. Solve the differential equations:

i) $2y(x+1)\frac{dy}{dx} = 4 + y^2$, given that $y = 2$ when $x = 3$.

Separating the variables,

$$\int \frac{2y}{4+y^2} dy = \int \frac{dx}{x+1}$$

$In(4+y^2) = In(x+1) + Ink \quad y = 2$ when $x = 3$

$In8 = In4 + Ink, \quad In2 = Ink, \quad k = 2$

$In(4+y^2) = In2(x+1)$ thus, $(4+y^2) = 2(x+1)$

To get $y = \sqrt{2(x-1)}$

ii) $y\cos^2 x\frac{dy}{dx} = \tan x + 2$, given that $y = 2$ when $x = \frac{\pi}{4}$

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$$\int y\,dy = \int \frac{\tan x + 2}{\cos^2 x}\,dx$$

$\int y\,dy = \int \tan x \sec^2 x\,dx + 2\int \sec^2 x\,dx$, but $d(\tan x) = \sec^2 x\,dx$

$$\frac{y^2}{2} = \frac{\tan^2 x}{2} + 2\tan x + c$$

given that $y = 2$ when $x = \dfrac{\pi}{4}$

$2 = \dfrac{1}{2} + 2 + c$ thus, $c = -\dfrac{1}{2}$

$\dfrac{y^2}{2} = \dfrac{\tan^2 x}{2} + 2\tan x - \dfrac{1}{2}$,

Thus $y^2 = \tan^2 x + 4\tan x - 1$

7. Sketch the curve $y = \dfrac{x^3}{9 - x^2}$, clearly stating the asymptotes.

Intercepts: when $y = 0$, $x = 0$ so, $(0,\ 0)$, thus curve crosses the axes at origin.

Vertical asymptotes: $9 - x^2 = 0$, so vertical asymptotes at $x = -3$, $x = 3$

To find the slant asymptotes: we perform long division:

$$\begin{array}{r|l} & -x \\ -x^2 - 9 & x^3 + 0x^2 + 0x + 0 \\ & -(x^3 - 9x) \\ \hline & 9x \end{array}$$

We get $y = -x + \dfrac{9x}{9 - x^2}$, thus as $x \to \pm\infty$, $y \to -x$, so the line $y = -x$ is the slant asymptote.

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For the turning points, we have $\frac{dy}{dx} = 0$,

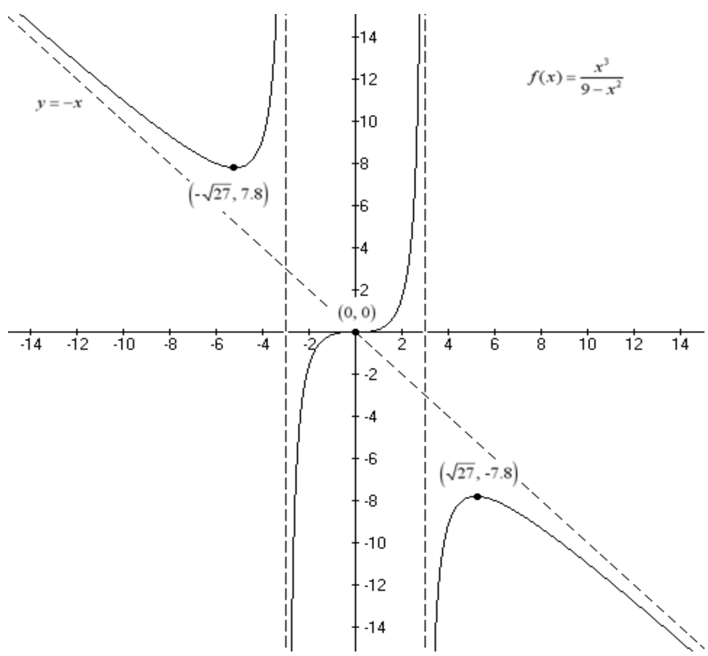
$$\frac{dy}{dx} = \frac{3x^2(9-x^2) - x^3. -2x}{(9-x^2)^2} = 0$$

$$27x^2 - 3x^4 + 2x^4 = 0$$

$x^2(27 - x^2) = 0$ these occur when $x = 0$, $x = \pm\sqrt{27}$,

$x = 0,\ y = 0$, $x = 3\sqrt{3},\ y = -7.794$, $x = -3\sqrt{3},\ y = 7.794$

i.e curve has a local minimum at $(-5.196\,,\ 7.794)$ and a local maximum at $(5.196\,,\ -7.794)$ and a negative point of inflexion at $(0,\ \ 0)$



8. Given that $y = In(x^2 + 2x + 3)$, show that $\frac{d^3y}{dx^3}(x^2 + 2x + 3) + (4x + 4)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$, hence, find Maclaurin's expansion of y showing the first four non-zero terms and approximate $In3.21$ correct to four decimal places.

Ssekyewa Edward Ddungu Ronald

$$y = In\left(x^2 + 2x + 3\right) \qquad \frac{dy}{dx} = \frac{2x+2}{x^2 + 2x + 3}$$

$$\left(x^2 + 2x + 3\right)\frac{dy}{dx} = 2x + 2$$

$$\left(x^2 + 2x + 3\right)\frac{d^2y}{dx^2} + (2x+2)\frac{dy}{dx} = 2$$

$$\left(x^2 + 2x + 3\right)\frac{d^3y}{dx^3} + (2x+2)\frac{d^2y}{dx^2} + \left(2x+2\right)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

$$\therefore \; \frac{d^3y}{dx^3}\left(x^2 + 2x + 3\right) + \left(4x+4\right)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$

Let $f\left(x\right) = In\left(x^2 + 2x + 3\right)$

$$f\left(0\right) = In3 \approx 1.0986$$

$$f'\left(0\right) = \frac{2}{3}$$

$$f''\left(0\right) = \frac{2 - 2\times\frac{2}{3}}{3} = \frac{2}{9}$$

$$f'''\left(0\right) = \frac{2 - 2\times\frac{2}{3} - 2\times\frac{2}{9} - 2\times\frac{2}{9}}{3} = -\frac{20}{7}$$

$$\Rightarrow \; f\left(x\right) = y = In3 + \frac{2}{3}x + \frac{2}{9}.\frac{x^2}{2!} + -\frac{20}{27}.\frac{x^3}{3!} + \ldots$$

$$= In3 + \frac{2}{3}x + \frac{x^2}{9} - \frac{10}{81}x^3 + \ldots$$

For $In3.21$ take $x = 0.1$

$$In3.21 \approx In3 + \frac{2}{3}\times 0.1 + \frac{0.1^2}{9} - \frac{10}{81}\times 0.1^3$$

$$= 1.16627$$

$\approx$ 1.1663 correct to 4 d.p.

9. The volume V of a cone is given by $V = \frac{1}{3}\pi r^2 h$. If the volume increases by $\frac{2}{3}\pi cm^3/\min$ and height increases by 0.03 $cm/\min$, find the rate of change of the radius when $r = 10cm$ and $h = 5cm$.

$V = \frac{1}{3}\pi r^2 h$ depends on r and t

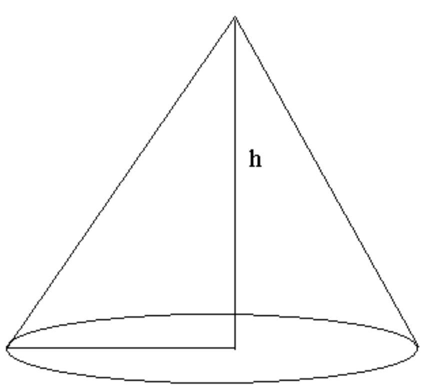
$$\Rightarrow \frac{dv}{dt} = \frac{\pi}{3} r^2 \frac{dh}{dt} + \frac{\pi}{3} h.2r\frac{dr}{dt}$$

$$\Leftrightarrow \frac{2\pi}{3} = \frac{\pi}{3}(100x0.03 + 2x10x5\frac{dr}{dt})$$

We get $2 - 3 = 100\frac{dr}{dt}$

$$\therefore \frac{dr}{dt} = -0.01cm/\min$$

10. Sand falls on to horizontal ground at the rate of 9m^3 per minute and forms a heap in the shape of a right circular cone with vertical angle 120^o. Show that $20\sqrt{3}$ seconds after sand begins to fall, the rate which the radius of the base of the pile is increasing is $\frac{\sqrt{3}}{\pi^{1/3}} m.\min^{-1}$.



$$\frac{dV}{dt} = 9 \; , \quad V = \frac{1}{3}\pi r^2 h$$

$$\frac{r}{h} = \tan 60^o \quad \Rightarrow \quad h = \frac{r}{\sqrt{3}} = \frac{\sqrt{3}}{3} r$$

$$\therefore V = \frac{\pi r^2}{3} . \frac{r}{\sqrt{3}} = \frac{\pi r^3}{3\sqrt{3}}$$

$$\frac{dr}{dt} = \frac{dV}{dt} . \frac{dr}{dV} = 9\frac{dr}{dV}$$

But $\frac{dV}{dr} = \frac{\pi r^2}{\sqrt{3}} \quad \Rightarrow \quad \frac{dr}{dV} = \frac{\sqrt{3}}{\pi r^2}$

$\therefore \quad \frac{dr}{dt} = 9.\frac{\sqrt{3}}{\pi r^2}$,

and since $\frac{dV}{dt} = 9$, then $V = 9t$

$\Rightarrow \frac{\pi r^3}{3\sqrt{3}} = \frac{9 \times 20\sqrt{3}}{60}$ thus $r^3 = 9x\frac{20\sqrt{3}}{60}x\frac{3\sqrt{3}}{\pi}$

$\therefore \quad r = \frac{3}{\pi^{1/3}} \qquad \because \quad \frac{dr}{dt} = \frac{9\sqrt{3}}{\pi}x\frac{1}{\frac{3^2}{\pi^{2/3}}} \quad = \quad \frac{\sqrt{3}}{\pi^{1/3}}m.\min^{-1}$

GEOMETRY

1. Prove that if two circles $C_1 : x^2 + y^2 + 2gx + 2fy + c = 0$ and $C_2 : x^2 + y^2 + 2Gx + 2Fy + C = 0$ are orthogonal then $2gG + 2fF = c + C$.

For orthogonal circles, $d^2 = r_1^2 + r_2^2$

For $C_1 : x^2 + y^2 + 2gx + 2fy + c = 0$,

$(x+g)^2 + (y+f)^2 = g^2 + f^2 - c$ thus $C_1(-g, -f)$, $r_1^2 = g^2 + f^2 - c$

For $C_2 : x^2 + y^2 + 2Gx + 2Fy + C = 0$,

$(x+G)^2 + (y+F)^2 = G^2 + F^2 - C$ thus $C_2(-G, -F)$, $r_2^2 = G^2 + F^2 - C$

So, substitute in $d^2 = r_1^2 + r_2^2$

$g^2 + f^2 - c + G^2 + F^2 - C = (-g+G)^2 + (-f+F)^2$

$g^2 + f^2 - c + G^2 + F^2 - C = g^2 - 2gG + G^2 + f^2 - 2fF + F^2$

Then $2gG + 2fF = c + C$. as required.

Ssekyewa Edward Ddungu Ronald

2. Find the equation of the line through the origin and concurrent with $2x-5y=3$ and $3x-4y=-2$.

Let $2x-5y-3+\lambda(3x-4y+2)=0$, where λ is a constant to be found.

But $x=0,\ y=0$

$\Rightarrow -3+2\lambda=0$, $\lambda=\frac{3}{2}$ or $\frac{2}{3}$

$\Rightarrow\ 2x-5y-3+\frac{3}{2}(3x-4y+2)=0$

$4x-10y-6+9x-12y+6=0$

$13x\ -22y=0$ or $22y-13x=0$

ALT. Let $2x-5y=3$, $3x-4y=-2$, solving simultaneously,

We get $x=-\frac{22}{7}$, $y=-\frac{13}{7}$

Grad of line $\frac{0--\frac{13}{7}}{0--\frac{22}{7}}=\frac{13}{22}$

$\therefore\ \frac{y-0}{x-0}=\frac{13}{22}\quad 22y-13x=0$

OR Let $2x-5y-3=\lambda(3x-4y+2)$

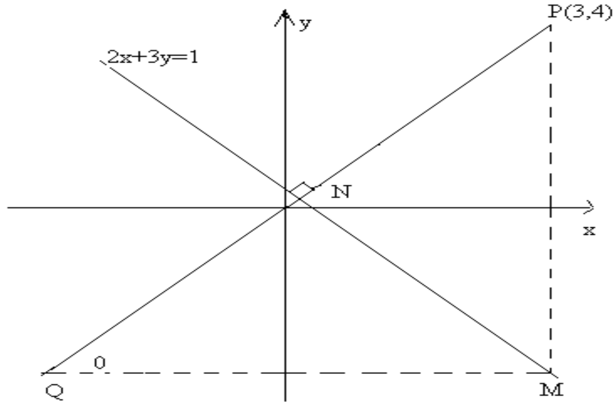
Put $x=0,\ y=0$

$\Rightarrow\ -3=2\lambda$, $\lambda=-\frac{3}{2}$ or $-\frac{2}{3}$

$2x-5y-3=-\frac{3}{2}(3x-4y+2)$

$22y-13x=0$

3. PN, the perpendicular from P(3, 4) to the line $2x+3y=1$ is produced to Q such that NQ = PN. Find the coordinates of Q.



Gradient of PQ = $\frac{3}{2}$ (perpendicular to $2x+3y=1$)

Equation of PQ is: $y-4=\frac{3}{2}(x-3)$ i.e $3x-2y-1$.

Solving this simultaneously with $2x+3y=1$, we obtain coordinates of N, $(\frac{5}{13},\frac{1}{13})$.

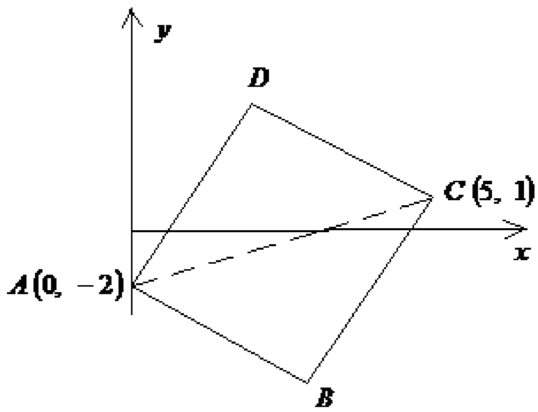
Let the coordinates of Q be (α,β), the coordinates of the midpoint of PQ i.e N are $\left(\frac{\alpha+3}{2},\frac{\beta+4}{2}\right)$

Thus; $\frac{\alpha+3}{2}=\frac{5}{13}$ giving $\alpha=-2\frac{3}{13}$

$\frac{\beta+4}{2}=\frac{1}{13}$ giving $\beta=-3\frac{11}{13}$

Coordinates of Q are $(-2\frac{3}{13},-3\frac{11}{13})$.

4. $ABCD$ is a square; A is the point $(0,\ -2)$ and C is the point $(5,\ 1)$, AC being the diagonal. Find the equations of the lines AB and BC.



AD & AB make angles of 45^o with diagonal AC.

So, the gradient of $AC = \frac{1-(-2)}{5-0} = \frac{3}{5}$, so if m_1 & m_2 are the gradients of AD & AB, then, $\tan 45 = \frac{m_1 - \frac{3}{5}}{1 + \frac{3}{5}m_1}$, where $m_1 = 4$ and so $m_2 = -\frac{1}{4}$ since AD & AB are perpendicular.

Equation of AD is; $\frac{y+2}{x-0} = 4$, $y = 4x - 2$

Equation of AB is; $\frac{y+2}{x-0} = -\frac{1}{4}$, $4y + x + 8 = 0$

Trigonometry

1. Prove that. $\frac{\tan A - 3\tan 3A}{\cot A - 3\cot 3A} = \frac{\tan A - 3\cot A}{\cot A - 3\tan A}$

From L.H.S $\frac{\tan A - 3\tan 3A}{\cot A - 3\cot 3A} = \frac{\tan A - \frac{3(3\tan A - \tan^3 A)}{1 - 3\tan^2 A}}{\cot A - \frac{3(\cot^3 A - 3\cot A)}{3\cot^3 A - 1}}$

$$= \frac{\dfrac{\tan A - 3\tan^3 A - 9\tan A + 3\tan^3 A}{1 - 3\tan^2 A}}{\dfrac{3\cot^3 A - 3\cot^3 A - \cot A + 9\cot A}{3\cot^2 A - 1}}$$

$$= \frac{-8\tan A}{1 - 3\tan^2 A} \cdot \frac{3\cot^2 A - 1}{8\cot A}$$

$$= \frac{\tan A - 3\cot A}{\cot A - 3\tan A} \text{ as the R.H.S}$$

2. Prove that in a triangle ABC,

i) $\dfrac{a+b-c}{a+b+c} = \tan\frac{1}{2}A\tan\frac{1}{2}B$

R.H.S $\tan\frac{1}{2}A\tan\frac{1}{2}B = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}} \times \sqrt{\dfrac{(s-a)(s-c)}{s(s-b)}}$

$$= \sqrt{\frac{(s-a)(s-b)(s-c)^2}{s^2(s-a)(s-b)}}$$

$$= \frac{s-c}{s}$$

$$= \frac{\frac{a+b+c}{2} - c}{\frac{a+b+c}{2}}$$

$$= \frac{a+b-c}{a+b+c} \text{ as L.H.S}$$

ALT: From L.h.S.

$$\frac{a+b-c}{a+b+c} = \frac{2R(\sin A + \sin B - \sin C)}{2R(\sin A + \sin B + \sin C)}$$

$$= \frac{2\sin\frac{1}{2}A\cos\frac{1}{2}A + 2\cos\frac{(B+C)}{2}\sin\frac{(B-C)}{2}}{2\sin\frac{1}{2}A\cos\frac{1}{2}A + 2\sin\frac{(B+C)}{2}\cos\frac{(B-C)}{2}}$$

$\cos\frac{B+C}{2} = \cos\left(90 - \frac{A}{2}\right) = \sin\frac{1}{2}A$

$\sin\frac{B+C}{2} = \cos\left(90 - \frac{A}{2} = \cos\frac{1}{2}A\right)$

$$= \frac{\sin\frac{1}{2}A\cos\frac{1}{2}A + \sin\frac{A}{2}\sin\frac{(B-C)}{2}}{\sin\frac{1}{2}A\cos\frac{1}{2}A + \cos\frac{A}{2}\cos\frac{(B-C)}{2}}$$

$$= \frac{\sin\frac{1}{2}A\left(\cos\frac{1}{2}A + \sin\frac{B-C}{2}\right)}{\cos\frac{1}{2}A\left(\sin\frac{1}{2}A + \cos\frac{B-C}{2}\right)}$$

$$= \frac{\sin\frac{1}{2}A\left(\sin\frac{B+C}{2} + \sin\frac{B-C}{2}\right)}{\cos\frac{1}{2}A\left(\cos\frac{B+C}{2} + \cos\frac{B-C}{2}\right)}$$

$$= \frac{\sin\frac{1}{2}A\left(2\sin\frac{B}{2}\cos\frac{C}{2}\right)}{\cos\frac{1}{2}A\left(2\cos\frac{B}{2}\cos\frac{C}{2}\right)}$$

$= \tan\frac{1}{2}A\tan\frac{1}{2}B$ as required.

ii) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A\cos B\cos C$

From the L.H.S

$$\sin^2 A + \sin^2 B + \sin^2 C = \frac{1}{2}(1-\cos 2A) + \frac{1}{2}(1-\cos 2B) + \sin^2 C$$

$$= 1 - \frac{1}{2}(\cos 2A - \cos 2B) + \left(1 - \cos^2 C\right)$$

$$= 2 - \frac{1}{2}(2\cos(A+B)\cos(A-B)) - \cos^2 C \quad : \cos(A+B) = -\cos C$$

$$= 2 - (-\cos C\cos(A-B)) - \cos^2 C$$

$$= 2 + \cos C(\cos(A-B) - \cos C)$$

$$= 2 + \cos C(\cos(A+B) + \cos(A-B))$$

$$= 2 + 2\cos A\cos B\cos C$$

3. Given that in a triangle MNB, $\tan A = \frac{n+b}{n-b}\tan\frac{1}{2}M$, where n, b & m are sides of the triangle MNB, show that $m = (n-b)\sec A\cos\frac{1}{2}M$.

$$\tan A = \frac{n+b}{n-b}\tan\frac{1}{2}M,$$

$$\tan^2 A = \frac{(n+b)^2}{(n-b)^2}\cdot\frac{\sin^2\frac{1}{2}M}{\cos^2\frac{1}{2}M}, \qquad \sec^2 A - 1 = \frac{(n+b)^2}{(n-b)^2}\cdot\frac{\sin^2\frac{1}{2}M}{\cos^2\frac{1}{2}M}$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\left(\sec^2 A-1\right)=(n+b)^2\sin^2\tfrac{1}{2}M$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\sec^2 A=(n+b)^2\left(1-\cos^2\tfrac{1}{2}M\right)+(n-b)^2\cos^2\tfrac{1}{2}M$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\sec^2 A=(n+b)^2-(n+b)^2\cos^2\tfrac{1}{2}M+(n-b)^2\cos^2\tfrac{1}{2}M$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\sec^2 A=(n+b)^2+\cos^2\tfrac{1}{2}M\left((n+b)^2-(n-b)^2\right)$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\sec^2 A=(n+b)^2-4nb\cos^2\tfrac{1}{2}M\ , \qquad \cos^2\tfrac{1}{2}M=\tfrac{1}{2}\left(1+\cos M\right)$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\sec^2 A=n^2+2nb+b^2-2nb\cos M-2bn$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\sec^2 A=n^2+b^2-2nb\cos M$$

$$(n-b)^2\cos^2\tfrac{1}{2}M\sec^2 A=m^2$$

Thus $m=(n-b)\sec A\cos\tfrac{1}{2}M$

VECTORS

1. A and B are points whose position vectors are $a=2i+5k$, $b=i+3j+k$ respectively, and the equations of the line L are $\dfrac{x-3}{2}=\dfrac{y-2}{2}=\dfrac{z-2}{1}$. Determine

i) the equation of the plane π which contains A and is perpendicular to L, and verify that B lies in the plane π.

Equation of the plane is of the form $r\,.\,n=d$ or $ax+by+cz=d$

Vector normal to the plane is the direction vector of the line $n=2i+2j+k$

Thus $a\,.\,n=d$ is $(2i+0j+5k).(2i+2j+k)$ $\therefore d=9$

Equation of the plane is $r\,.\,(2i+2j+k)=9$ or $2x+2y+z=9$

For the point B, $d=(i+3j+k).(2i+2j+k)=9$, thus since B satisfies the equation of the plane, then it lies there.

ii) the angle between the plane π above and the line $r=3i+j-3k+t\,(i-2j-4k)$.

Let θ be the angle between the plane and the line, thus the angle between the normal to the plane and the line is given by $n\,.\,d=|n|\,|d|\sin\theta$

Thus $n\,.\,d = (2i + 2j + k).(i - 2j - 4k) = -6$

$|n| = \sqrt{4+4+1} = \sqrt{9} = 3$

$|d| = \sqrt{1+4+16} = \sqrt{21}$

$\sin\theta = \dfrac{-6}{3\sqrt{21}} = -\dfrac{2}{\sqrt{21}}$

2. Find the point of intersection of the lines $x = \dfrac{y+1}{2} = \dfrac{z-5}{-1}$ and $\dfrac{x-1}{-1} = \dfrac{y+3}{-3} = \dfrac{z-4}{1}$

Let $x = \dfrac{y+1}{2} = \dfrac{z-5}{-1} = \lambda$, $\Rightarrow x = \lambda,\ y = 2\lambda - 1,\ z = -\lambda + 5$

Let $\dfrac{x-1}{-1} = \dfrac{y+3}{-3} = \dfrac{z-4}{1} = \mu \Rightarrow x = -\mu + 1,\ y = -3\mu - 3,\ z = \mu + 4$

Solve to obtain $\mu = -4,\ \lambda = 5$, check to see that they do intersect,

Hence the point is $(5, 9, 0)$.

b) Prove that the vectors $a = 3i + j - 4k$, $b = 5i - 3j - 2k$ and $c = 4i - j - 3k$ are coplanar.

For coplanar vectors a, b, c, $a = \lambda b + tc$, where $\lambda \,\&\, t$ are scalars.

So, $\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} = \lambda\begin{pmatrix} 5 \\ -3 \\ -2 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$, thus $\begin{array}{l} 5\lambda + 4t = 3 \\ -3\lambda - t = 1 \\ -2\lambda - 3t = -4 \end{array}$ solving to get $t = 2,\ \lambda = -1$

Check using equation (iii) L.H.S = $2 + -6 = -4$ = R.H.S

Since the value of $\lambda \,\&\, t$ are consistent, then the vectors are coplanar.

c) The vector equation of a line is given by $r = 17i + 2j - 6k + \lambda(-9i + 3j + 9k)$. Find:

i) A vector parallel to this line.

Note: Parallel lines have the same direction ratios.

$-9:3:9=-3:1:3$, so either $-3i+j+3k$ or $3i-j-3k$ are parallel to the line.

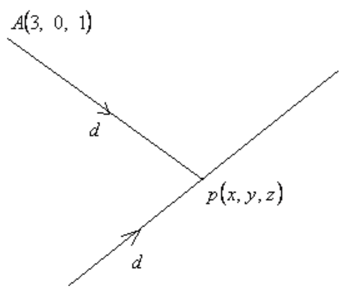
ii) State one point through which the line passes.

$(17, 2, -6)$

3i) Find the perpendicular distance of the point $(3,0,1)$ from the line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z}{12}$

$\frac{x-1}{3}=\frac{y+2}{4}=\frac{z}{12}$; $(3, 0, 1)$

so let $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z}{12}=\lambda$, $x=1+3\lambda, y=4\lambda-2, z=12\lambda$



$AP=OP-OA$

$$=\begin{pmatrix}3\lambda+1\\4\lambda-2\\12\lambda\end{pmatrix}-\begin{pmatrix}3\\0\\1\end{pmatrix}=\begin{pmatrix}3\lambda-2\\4\lambda-2\\12\lambda-1\end{pmatrix}$$

AP is perpendicular to $d=3i+4j+12k$, thus, $AP.d=0$

$$\begin{pmatrix}3\lambda+1\\4\lambda-2\\12\lambda\end{pmatrix}.\begin{pmatrix}3\\4\\12\end{pmatrix}=0,\quad 9\lambda-6+16\lambda-8+144\lambda-12=0,\quad \lambda=\frac{2}{13}$$

$$AP = \begin{pmatrix} \frac{6}{13}-2 \\ \frac{8}{13}-2 \\ \frac{24}{13}-1 \end{pmatrix} = \begin{pmatrix} -\frac{20}{13} \\ -\frac{8}{13} \\ \frac{11}{13} \end{pmatrix}$$, the perpendicular distance

$$d = |AP| = \sqrt{\left(-\frac{20}{13}\right)^2 + \left(-\frac{18}{13}\right)^2 + \left(\frac{11}{13}\right)^2} = 2.23607$$

ii) Find the angle between the line $x-1=\frac{y-2}{0}=\frac{z-3}{3}$ and the plane $3x+2y+z=1$.

$d = i + 0j + 3k$, $3x+2y+z=1$; $n = 3i + 2j + k$

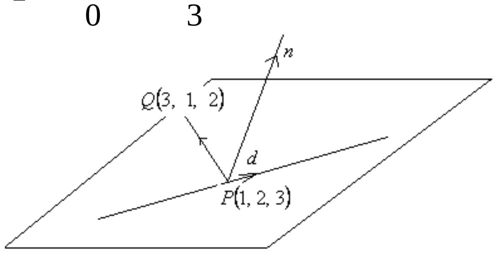
Let θ be the angle between the line and plane is given by $n \,.\, d = |n|\,|d| \sin\theta$

$$n \,.\, d = (3i + 2j + k).(i + 0j + 3k) = 6$$

$$|n| = \sqrt{1^2 + 3^2} = \sqrt{10}, \quad |d| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$$

$$\sin\theta = \frac{6}{\sqrt{10}\sqrt{14}} = \frac{6}{\sqrt{140}}, \quad \theta = 30.47^o$$

iii) Determine in dot product form the equation of a plane containing the point $(3,1,2)$ and the line $x-1=\frac{y-2}{0}=\frac{z-3}{3}$



$PQ = (3i + j + 2k) - (i + 2j + 3k) = i - j - k$

Let the normal have direction ratios $a:b:c$

$n \,.\, PQ = 0$, thus, $(ai + bj + ck).(i - j - k) = a - b - c = 0$

Also, $n \,.\, d = 0$, $(ai + bj + ck).(i + 0j + 3k) = a + 3c = 0$

So, $a = -3c$, $b = -4c$, thus $a:b:c = -3c:-4c:c = -3:-4:1$

Equation is $r \,.\begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.\begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix}$ thus $r \,.\begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix} = -8$

TRY:

1. The points A and B have coordinates $(2,\ 1,\ 1)$ and $(0,\ 5,\ 3)$ respectively. Find the equation of the line AB in parametric form. If C is the point $(5,\ -4,\ 2)$, find the coordinates of the point D on AB such that CD is perpendicular to AB.

2. Determine the equation of the plane through the points $A(1,\ 1,\ 2)$, $B(2,\ -1,\ 3)$ and $C(-1,\ 2,\ -2)$.

   b) A line through the point $D(-13,\ 1,\ 2)$ and parallel to the vector $12\mathbf{i}+6\mathbf{j}+3\mathbf{k}$ meets the plane in (a) at point E. Find:

   i) the coordinates of E.

   ii) The angle between the line and the plane.

3. Solve the simultaneous equations:
$$z_1 + z_2 = 8$$
$$4z_1 - 3iz_2 = 26 + 8i$$

4. Use De'Moivres theorem to find the modulus and argument of $\dfrac{(3+9i)^9(1+i)^3}{(-1-i)}$

5. Prove that in a triangle ABC,

   i) $a\cos ec\frac{1}{2}A = (b+c)\sec\frac{1}{2}(B-C)$, find C if $a = 5.3, b+c = 11.8, and A = 46^o$

   ii) $\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$, find the largest angle and area of the triangle in which $a = 10.6, b = 11.8, And\ c = 15.6$

THANK YOU AND KEEP WORKING HARD

OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2023

ALGEBRA

1. (a) Find the square root of $14 + 6\sqrt{5}$

(b) Given $y = x + \frac{1}{x}$, Solve for x in the equation $4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0$

(c) Find the first three terms of the Binomial expansion of $\sqrt{(1+x)(1+x^2)}$

(d) Given that the equation $x^2 + 3x + 2 = 0$ has roots k and l, find the equation whose roots are $\frac{k}{l^2}$ and $\frac{l}{k^2}$

(OLAN)

2. (a) Prove by induction that for all positive integer$\sum_{r=1}^{n}(3r+1)(r+2) = n(n+2)(n+3)$

(b) Prove by induction that for all positive odd integers, n, $f(n) = 4^n + 5^n + 6^n$ is divisible by 15

(c) Use Demoivre's theorem to prove that $sin5\theta = 5sin\theta - 20sin^3\theta + 16sin^5\theta$

(d) Hence or otherwise, find the distinct roots of the equation $2 + 10x - 40x^2 + 32x^5 = 0$ giving your answer correct to 3 dps where appropriate. **(JINJA COLLEGE)**

3. (a) The sum of n terms of the sequence is $S_n = 2^{2n} - n$ where n a natural numbers is 61. Find the first three terms of the sequence.

(b)From a class of 14 boys and 10 girls, 10 students are to be selected for a competition in which 5 boys and 5girls or 2girls and 8boys are to go for it. In how many ways can they be selected?

(c) The roots p and q of a quadratic equation are such that $p^3 + q^3 = 4$ and $pq = \frac{1}{2}(p^3 + q^3) + 1$. Find a quadratic equation with integral coefficients whose roots are p^6 and q^6.

(d). Mary operates an account with a bank which offers a compound interest of 5% per annum. She opened the account at beginning of 2019 with shs 800,000 and continue to deposit the same amount at beginning of every year. How much will she receive at the end of 2022. If she made no withdrawal with in this period?

(ST CHARLES LWANGA BUKERERE)

4. (a) Solve the inequality $\frac{x+3}{x-2} \geq \frac{x+1}{x-2}$

(b) Given the curve $y = \frac{x^2+x-2}{x^3-7x^2+14x-8}$

(i) Give the coordinates of the hole

(ii) Find the equations of the asymptotes

(iii) Determine the turning points and their nature

(iv) Find the intercepts and sketch the curve.

(ST MICHEAL S.S SONDE)

TRIGONOMETRY

5. (a) Given the function, $f(x) = \frac{3}{13+6sinx-5cosx}$ use the substitution $t = tan\left(\frac{x}{2}\right)$, to show that $f(x)$ can be written in the form $\frac{3(1+t^2)}{2\,(3t+1)^2+6}$

(b) Given that $cot^2\theta + 3cosec^2\theta = 7$, show that $tan\theta = \pm 1$

(c) (i) Express the function $y = 3cosx - \sqrt{3}sinx$ in the form $Rcos(\theta + \alpha)$where R is a constant and $0 \leq \alpha \leq 2\pi$

(ii) Hence find the coordinates of the minimum point of y

(iii) State the values of x at which the curve cuts the $x - axis$

(CODE HIGH SCHOOL)

6. (a) Show that $cos3\theta = 4cos^3\theta - 3cos\theta$. Hence if $cos\theta = \frac{1}{2}(a + \frac{1}{a})$, prove that $cos3\theta = \left(a^3 + \frac{1}{a^3}\right)$

(b) Given that A, B, and C are angles of a triangle, prove that; $Sin^2\,{}^{A}/_{2} + Sin^2\,{}^{B}/_{2} + Sin^2\,{}^{C}/_{2} = 1 - 2Sin\,{}^{A}/_{2}\,Sin\,{}^{B}/_{2}\,Sin\,{}^{C}/_{2}$

(c) Given that $Sin(\theta + \alpha) = a$ and $Sin(\theta + \beta) = b$. Show that $Cos2(\alpha - \beta) - 4abCos(\alpha - \beta) = 1 - 2a^2 - 2b^2$.

(JINJA PROGRESSIVE)

7. (a) Given that $cos45^0 = \frac{1}{\sqrt{2}}$. Show without using a calculator or tables that $Sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2+\sqrt{2}}$

(b) If $cos\alpha - cos\beta = \frac{2}{3}$ and $sin\alpha - sin\beta = \frac{5}{6}$, find the value of;

(i) $Sin\frac{1}{2}(\alpha + \beta)$ (ii) $Cos(\alpha + \beta)$

I(i) Given that A,B and C are angles of $\frac{a^2+b^2-c^2}{a^2-b^2+c^2} = tanBcotC$.

(ii) Find all the sides of a triangle ABC whose area is $1008cm^2$ and $a = 65cm,\ b + c = 97cm$.

(MENTHA HIGH SCHOOL)

8. (a) Given that X, Y, Z are angles of a triangle, Prove that $tan\left(\frac{X-Y}{2}\right) = \left(\frac{x-y}{x+y}\right)Cot\left(\frac{Z}{2}\right)$, hence solve the triangle if $x = 9cm, y = 5.7cm\ and\ Z = 57^0$.

(b) Prove that $\frac{Cos11^0+sin11^0}{Cos11^0-Sin11^0} = tan56^0$

(c) In triangle ABC, $AB = x - y, BC = x + y\ and\ CA = x$ Show that $CosA = \frac{x-4y}{2(x-y)}$

(d)(i) Show that $tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{5}\right) = tan^{-1}\left(\frac{7}{9}\right)$.

(ii) Prove that $Sin(2sin^{-1}x + cos^{-1}x) = \sqrt{1 - x^2}$

(GAYAZA ROAD TRIANGLE KIWENDA)

ANALYSIS

9. (a) Use the method of small changes to find the value of $\frac{1}{\sqrt{0.97}}$ correct to 3dps

(b) Evaluate $\int_0^1 \frac{8x-8}{(x+1)^2(x-3)^2}\,dx$

(c)(i) Given that $f(x) = \frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$, Express $f(x)$ into partial fractions

(ii) Hence evaluate $\int_4^5 f(x)\,dx$

(d) Evaluate $\int_0^{\frac{\pi}{4}} \frac{4}{1+cos2x}\,dx$

(MUKONO KINGS)

10(a)(i). On the same axes, sketch the curve $y = x(x+2)$ and $y = x(4-x)$.

(ii). Find the area enclosed by the two curves in $a(i)$ above

(iii). Determine the volume generated when the area enclosed by the two curves in $a(i)$ above is rotated about the $x-axis$.

(b) Evaluate $\int_2^6 \frac{\sqrt{x-2}}{x}\,dx$

(c) A match box consists of an outer cover open at both both ends into which a rectangular box without a top. The length of the box is 1.5 times the width. The thickness of the material is negligible and the volume of the match box is $25cm^3$. If the width is xcm, find interms of x the area of the material used. Hence show that if the least area of the material is to be used to make the box then the length should be approximately $3.7cm$

(OLAM)

11.(a) In order to post a parcel, the sum of the circumference of a cylindrical parcel and its height should add up to $6cm$. Find the dimensions of the largest parcel that can be accepted.

(b) A sample of bacteria in a sealed container is being studied. The number of bacteria, P in thousands, is given by the differential equation $(1+t)\frac{dp}{dt} + p = (1+t)\sqrt{t}$ where t the time in hours after the start of the study is. Initially, there are exactly 5,000 bacteria in the container.

(i) Determine, according to the differential equation, the number of bacteria in the container $8hours$ after the start of the study.

(ii) Find, according to the differential equation, the rate of change of the number of bacteria in the container 4 $hours$ after the start of the study.

(NAMRUTH HIGH SCHOOL)

12. (a) Given that $y = \log_e \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$, prove that $\frac{dy}{dx} = -secx$

(b) Solve the differential equation $x\frac{dy}{dx} = 2x - y$

(c) In an agricultural plantation the proportion of the total area that has been destroyed by a bacterial disease is x. The rate of the destruction of the plantation is proportional to the product of the proportion already destroyed and that not yet. It was initially noticed that half of the plantation had been destroyed by the disease and that at this rate another quarter of the plantation would be destroyed in the next $6 hours$.

(i) Form a differential equation relating x and time t

(ii) Calculate the percentage of the population destroyed 12 $hours$ after the disease was noticed.

(ST JAMES BUDDO)

VECTORS

13. (a) Find the angle $\alpha = \langle BAC$ of the triangle ABC whose vertices are $A(1,\ 0,\ 1)$, $B(2,-1,\ 1)$ and $C(-2,\ 1,\ 0)$.

(b) The planes $P_1 and\ P_2$ are respectively given by the equations $r = 2i + 4j - k + \lambda(i + 2j - 3k) + \mu(-i + 2j + k)$ and $r.(2i - j + 3k) = 5$ where λ and μ are scalar parameters. Find;

(i) The Cartesian equation for plane, P_1 .

(i) To the nearest degree, the acute angle between P_1 and P_2

(iii) The coordinates of the point of intersection of the plane, P_1 and the line $\frac{x-1}{5} = \frac{y-3}{-3} = \frac{z+3}{4}$

(WAKISO HILLS)

14(a) Given the line $r = (3 + 2\mu)i + (1 - \mu)j + (-2 + 2\mu)k$. Find the;

(i) Value of d if the line is in the plane $r.(i - 2j - 2k) = d$.

(ii) Distance of the point (3,1,7) from the line.

(b) Given the points $A(2,-5,3)\ and\ B(7,0,-2)$, find the coordinates of point C which divides AB externally in the ratio 3: 8.

(c) Show that the points $P(1,2,3), R(3,8,1)\ and\ T(7,20,-3)$ are collinear.

(PRIDE COLLEGE)

15(a) Find the equation of the plane which contains a point $A(2,1,-2)$and is parallel to the plane $x - y - 4z = 3$

(b) If a line $\frac{x}{2} = \frac{y+3}{0} = \frac{z-1}{1}$ intersects with the plane in (a) above, find the;

(i) Point of intersection.

(ii) The angle between the line and the plane.

(WHITE ANGELS HIGH SCHOOL)

16 (a). Find the vector equation of the line of intersection between the planes $r.\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} = 6$ and $r.\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 4$.

(b) Using the dot product only, find the equation of the plane containing points $A(0,1,1), B(2,1,0)$ and $C(-2,0,3)$.

(c) A straight line joining the points $(2,1,4)$ and $(a-1,4,\ -1)$ is parallel to the line joining points $(0,2,\ b-1)$ and $(5,3,\ -2)$. Find the values of a and b.

(TALENTS COLLEGE)

COORDINATE GEOMETRY

17.(a) The points A, B, and C have coordinates $A(-3,\ 2),\quad B(-1,-2)$ and $C(0,\ n)$, where n is a constant. Given that $\overline{BC} = \frac{1}{5}\overline{AC}$, Find possible values of n.

(b) A straight line L, passes through the point (-2, 1) and makes an angle of 45^0 with the horizontal.

(i) Find the equation of line L.

(ii) Given that the line L, intersects the $x-axis$ at A and the $y-axis$ at B. Find the distance AB.

(c) Find the coordinates of the circumcenter of the triangle ABC with vertices $A(3,2), B(1,4)$ and $C(5,4)$.

(d) Given that $A(0,-5), B(-7,2)$ and $C(2,11)$ are vertices of a parallelogram $ABCD$, find the coordinates of the point D.

(MAKERERE HIGH SCHOOL)

18.(a) Find the equation of the circle whose centre is at (5,4) and touches the line joining (0,5) and (4,1)

(b) A circle that passes through the points $A(3,4)$ and $B(6,1)$ and the equation of the tangent to this circle at A is the line $2y = x + 5$. Find;

(i) The coordinates of the centre of the circle.

(ii) The radius of the circle.

(iii) The equation of the circle.

(c) If $y = mx$ is a tangent to a circle $x^2 + y^2 + 2fy + c = 0$ prove that $c = \frac{f^2m^2}{1+m^2}$. Hence find the equation of the tangents from origin to the circle $x^2 + y^2 - 10y + 20 = 0$.

(d) Show that the circles whose equations are $x^2 + y^2 - 4y - 5 = 0$ and $x^2 + y^2 - 8x + 2y + 1 = 0$ cut orthogonally.

(QUEENS S.S)

19.(a) P and Q are two points whose coordinates are $(at^2, 2at), (\frac{a}{t^2}, \frac{-2a}{t})$ respectively and S is a point$(a, 0)$. Show that $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$

(b) Prove that $x = 3t^2 + 1$ and $2y = 3t + 1$ are parametric equations of a parabola. Find its Vertex, Focus and length of latus rectum.

(c) The point $P(at^2, 2at)$ is on the parabola $y^2 = 4ax$. The chord OQ passes through the origin$O$. The tangent at P is parallel to the chord OQ. The tangents to the parabola at P and Q meet at a point R. Determine the coordinates of points Q and R in terms of a and t.

(ST JOHNS NTEBETEBE)

20.(a) Show that $x^2 + 2y^2 + 6x - 8y = 7$ is an ellipse and hence determine its centre and eccentricity

(b) Points S and S' are the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. Find the coordinates of S and S'

(c) The conic section below has eccentricity $e < 1$ and equation $\frac{x^2}{9} + y^2 = 1$. Find the value of e

(d).If the line $y = mx + c$ is a tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $c^2 = b^2 + a^2m^2$.

Hence determine;

(i) Equations of the four common tangents to the ellipses $\frac{x^2}{23} + \frac{y^2}{3} = 1$ and $\frac{x^2}{14} + \frac{y^2}{4} = 1$

(ii) The equations of the tangents at the point $(-3,3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(PAUL MUKASA HIGH SCHOOL)

21(a). Find the equation of the tangent to the hyperbola $x^2 - 9y^2 = 1$ at $P(sec\beta, \frac{1}{3}tan\beta)$

(b) The tangent at any point $P\left(ct, \frac{c}{t}\right)$ on the hyperbola $xy = c^2$ meets x and y at A and B respectively. O is the origin.

(i) Prove that $AP = PB$

(ii) Prove that the area of triangle AOB is constant

(iii) If the hyperbola is rotated through an angle of -45^0 about O, find the new equation of the curve.

(GOOD CHOICE HIGH SCHOOL)

END

OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS SEMINAR QUESTIONS 2022

ALGEBRA

1. (a) Without using mathematical tables or calculators, find the value of $\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{8\sqrt{5}}$.

(b) Given that $2x^2+7x-4$, x^2+3x-4, and $7x^2+ax-8$ have a common factor, find the;

(i) Factors of $2x^2+7x-4$ and x^2+3x-4.

(ii) Value of a in $7x^2+ax-8$.

(c) A man deposits shs 150,000 at the beginning of every year in a microfinance bank with the understanding that at the end of seven years, he is paid back his money with 5% per annum compound interest. How much does he receive?

(d) Given that the roots of the equation; $x^2-bx+c=0$ are $\sqrt{\propto}$ and $\sqrt{\beta}$. Show that;

(i) $\propto+\beta=b^2-2c$

(ii) $\propto^2+\beta^2=\left(b^2-2c-\sqrt{2}c\right)\left(b^2-2c+\sqrt{2}c\right)$

2. (a) Find the solution set for which $\log_2 x-\log_x 4\leq 1$.

(b) Solve the simultaneous equations; $2a-3b+c=10$,

$$a+4b+2c+3=0,$$
$$5a-2b-c=7$$

(c) A geometric progression has the first term 10 and sum to infinity of 12.5. How many terms of the progression are needed to make a sum which exceeds 10?

(d) Given that the equations $y^3-2y+4=0$ and $y^2+y+c=0$ have a common root, show that $c^3+4c^2+14c+20=0$.

3. (a) Simplify $(2+5i)^2+5\left(\frac{7+2i}{3-4i}\right)-i(4-6i)$ expressing your answer in the form $a+bi$.

(b) The roots of the equation $3x^2+2x-5=0$ are $\propto$ and β. Find the value of $\propto^4+\beta^4$.

(c) Solve the equation $\sqrt{x+5}+\sqrt{x+21}=\sqrt{6x+40}$.

(d) Given that $\log_5 21=m$ and $\log_9 75=n$, show that $\log_5 7=\frac{1}{2n-1}(2nm-m-2)$

4. (a) Expand $(1-x)^{\frac{1}{3}}$ as far as the term in x^3. Use your expansion to deduce $\sqrt[3]{24}$ correct to three s.f.

(b) In the expansion of $(1+ax)^n$, the first three terms are $1-\frac{5}{2}x+\frac{75}{8}x^2$. Find n and a , state the range of values for which the expansion is valid.

(c) Determine the term independent of x in the binomial expansion of $\left(\frac{3}{x^2} - 2x\right)^6$.

(d) Find the coefficient of x^{17} in the expansion of $\left(x^3 + \frac{1}{x^4}\right)^{15}$

ANALYSIS

5. (a) If $y = tan^{-1}\left(\frac{ax-b}{bx+a}\right)$,show that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

(b) Differentiate $cos(x^2e^x)$ with respect to x.

(c) Differentiate cos^2x with respect to x from first principles.

(d) Given that $x = \frac{3t-1}{t}$ and $y = \frac{t^2+4}{t}$,

(i) Show that $\frac{d^2y}{dx^2} = 2t^3$

(ii) Determine the area of the largest rectangular piece of land that can be enclosed by $100m$ of fencing if part of an already existing wall is used.

6. (a) Solve the differential equation $\frac{dy}{dx} + \frac{2xy}{x^2+1} - x = 0$

(b) The gradient of a certain curve is given by kx. If the curve passes through the point (2,3) and the tangent at this point makes an angle of $tan^{-1}(6)$ with the positive direction of the x-axis, find the equation of the curve.

(c) A research to investigate the effect of a certain chemical on a virus infection, the research revealed that the rate at which the virus population is destroyed is directly proportional to the population at that time. Initially , the population was p_o. At t *months* later, it was found to be p.

(i) Form a differential equation connecting p and t

(ii) Given that the virus population reduced to one third of the initial population in 4 *months*, solve the equation in c(i) above.

7. (a) Partialise fully, $f(x) = \frac{x^4+x^3-6x^2-13x-6}{x^3-7x-6}$. Hence $\int f(x)dx$ from 4 to 5.

(b)(i) On the same axes, sketch the curve $y = x(x+2)$ and $y = x(4-x)$

(ii) Find the area enclosed by the two curves in b(i) above

(iii) Determine the volume generated when the area enclosed by the two curves in b(ii) above is rotated about the x-axis.

8. (a) Find the integral $\int x cos^2x\, dx$

(b)Find the area enclosed between the curve $y = x(x-1)(x-2)$ from $x = 0$ *to* $x = 2$

(c) A conical vessel whose height is $10 metres$ and radius of the base $5m$ is being filled with water at a uniform rate of $1.5m^3min^{-1}$. Find the rate at which the level of the water in the vessel is rising when the depth is $4m$

(d) Find the area enclosed by the curve $y = x - \frac{1}{x}$, the $x - axis$ and the line $x = 2$.

TRIGONOMETRY

9. (a) Solve the equation $3cos4\theta + 7cos2\theta = 0\ for\ 0^0 \le \theta \le 180^0$

(b) Express $10sinxcosx + 12\ cos2x$ in the form $Rsin(2x{+}\propto)$. Hence find the maximum value of $10sinxcosx + 12\ cos2x$.

(c) Prove that $\frac{cos11^0+sin11^0}{cos11^0-sin11^0} = tan56^0$

(d) In any triangle ABC, prove that $SinB + SinC - SinA = 4cos{}^{A}/_{2}\,Sin{}^{B}/_{2}\,Sin{}^{C}/_{2}$.

10. (a) Find the values of x lying between $-180^0\ and\ 180^0$ that satisfy the equation $10sin^2x + 10sinxcosx = cos^2x + 2$

(b) If $\frac{Sin16\theta cos2\theta - cos6\theta sin12\theta}{cos4\theta cos2\theta + sin6\theta sin8\theta} = tanm\theta$, where m is a constant, find the value of m.

(c) Find all the solutions to $2sin3\theta = 1$ for θ between $0^0\ and\ 360^0$. Hence find the solutions for $8x^3 - 6x + 1 = 0$

(d)Show that in any triangle ABC, $\frac{a^2-b^2}{c^2} = \frac{Sin(A-B)}{Sin(A+B)}$

11. (a) Using the t-formulae or otherwise prove that $1 + sec2\theta = tan2\theta cot\theta$

(b) Given that $y = \frac{sinx-2sin2x+sin3x}{sinx+2sin2x+sin3x}$;

(i) Prove that $y + tan^2\left(\frac{x}{2}\right) = 0$,

(ii) And hence express the exact value of $tan^2 15^0$ in the form $p + q\sqrt{r}$ where $p, q\ and\ r$ are integers.

(iii) Hence, find the value of x in the range $0^0 \le x \le 360^0$ for which $2y + sec^2\left(\frac{x}{2}\right) = 0$

12. (a)Given that $45^0 = \frac{1}{\sqrt{2}}$. Show without using a calculator or tables that $Sin\left(292\frac{1}{2}^0\right) = -\frac{1}{2}\sqrt{2+\sqrt{2}}$

(b) Given that $P = 2cos2x + 3cos4x$ and $q = 2sin2x + 3sin4x$;

(i) Find the greatest and least value of $p^2 + q^2$.

(ii) Given that $p^2 + q^2 = 19$, find x for $0^0 \le x \le 90^0$

(iii) Using the result in (ii) above, and without using a calculator or Mathematical tables, show that $pq = \frac{-5\sqrt{3}}{4}$.

VECTORS

13. (a) Line A is intersection of two planes whose equations are $3x - y + z = 2$ *and* $x - 5y + 2z = 6$. Find the Cartesian equation of the line

(b) Given that line B is perpendicular to the plane $3x - y + z = 2$ and passes through the point C(1,1,0), find the;

(i) Cartesian equation of the line B

(ii) Angle between the line B and line A in (a) above.

(c) The point $A(2,-1,0)$, B(-2, 5, -4) and C are on a straight line such that $3\overrightarrow{AB} = 2\overrightarrow{AC}$. Find the coordinates of C.

14. (a)The points P, Q, R have position vectors $2\boldsymbol{a} - 5\boldsymbol{b}, 5\boldsymbol{a} - \boldsymbol{b}, and\ 11\boldsymbol{a} + 7\boldsymbol{b}$ respectively. Show that P, Q and R are collinear and state the ratio $PQ : QR$.

(b) Calculate the perpendicular distance from the point $(1, -2, 3)$ from the line with equation $\boldsymbol{r} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k})$.

(c) Determine the equation of a plane through the point $(1, -3, 2)$ and contains the vectors $\boldsymbol{i} - 3\boldsymbol{j} + 3\boldsymbol{k}$ and $-\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$.

15. (a) The point $C(a, 4, 5)$ divides the line joining $A(1,2,3)$ and B (6,7,8) in the ratio $\lambda : 3$. Find a and λ

(b) Find the equation of a plane which contains the line $\frac{x-1}{2} = \frac{y+4}{-3} = \frac{z+1}{-1}$ and passes through the point $(2, 3, -1)$

(c) Show that the lines L_1, with vector equation $\boldsymbol{r} = \binom{2}{5} + \lambda\binom{2}{-3}$ and L_2, vector equation $r = \binom{3}{-3} + \mu\binom{3}{2}$ are perpendicular and find the position vector of their point of intersection.

16. (a) Find the length of the perpendicular distance from $A(4,3,5)$ to the plane $6x - y + 2z = 14$

(b) Find the foot of the perpendicular drawn from the point $(2, -1, 5)$ to the line $\frac{x-11}{10} = \frac{y+2}{-2} = \frac{z+5}{-11}$.

(c) Find the angle between the plane $x - 2y + z = 20$ and the line $\frac{2-x}{-3} = \frac{y+1}{4} = \frac{2-z}{-12}$

(d) $PQRS$ is a quadrilateral $P(1, -2), Q(4, -1), R(5,2)$ and $S(2,1)$. Show that the quadrilateral is a rhombus.

COORDINATE GEOMETRY

17. (a) Find the locus of a point which moves such that the ratio of its distance from the point $A(2,4)$ to its distance from the point $B(-5,3)$ is $2:3$

(b) A point P moves such that its distance from two points $A(-2,0)$ and $B(8,6)$ is in the ratio $AP:PB = 3:2$. Show that the locus of P is a circle.

(c) Find the locus of the point $P(x, y)$ which moves such that its distance from the point $A(5,3)$ is twice its distance from $x = 2$

(d) Find the centroid of the triangle whose sides are given by the equations $x + y = 11$, $y = x - 1$ and $3y = x - 3$.

18. (a) find the equation of the circle whose end diameter is the line joining the points $A(1,3)$ and $B(-2,5)$.

(b) A circle whose centre is in the first quadrant touches the x- and y-axes and the line $8x - 15y = 20$.

Find the;

(i) Equation of the circle

(ii) Point at which the circle touches the y-axis

(c) If the x-axis and y-axis are tangents to the circle$x^2 + y^2 + 2gx + 2fy + c = 0$, Show that $c = g^2 = f^2$

19. (a) ABCD is a square inscribed in a circle $x^2 + y^2 - 4x - 3y = 36$, Find the length of diagonals and the area of the square

(b) Find the coordinates of the foot of the perpendicular from the point (2,-6) to the line $3y - x + 2 = 0$.

(c) A circle touches both x-axis and the line $4x - 3y + 4 = 0$. Its centre is in the first quadrant and lies on the line $x - y - 1 = 0$. Prove that its equation is $x^2 + y^2 - 6x - 4y + 9 = 0$

(d) ABCD is a rhombus such that the coordinates $A(-3,-4)$ and $C(5,4)$. Find the equation of the diagonal BD of the rhombus. If the gradient of side BC is 2 obtain the coordinates of B and D, Prove that the area of the rhombus is $21\frac{1}{3}sq.Units.$

20. (a) Show that the equation $y^2 - 4y = 4x$ represents a parabola; hence determine the focus, vertex and directrix.

(b) A tangent from the point $T(t^2, 2t)$ touches the curve $y^2 = 4x$. Find;

(i) The equation of the tangent

(ii) The equation of the line L parallel to the normal at $T(t^2, 2t)$ and passes through (1,0)

(iii) The point of intersection, X, of the line L and the tangent

(c) A point $P(x, y)$ is equidistant from X and T in (b) above, Show that the locus of P is $t^3 + 3t - 2(xt + y) = 0$

(d) Find the equations of the tangents to the parabola $y^2 = 6x$ which pass through the point (10,-8)

END

S.6 APPLIED MATHEMATICS PAPER 2 REVISION QUESTIONS

1. A body of mass 10 kg rests on a smooth horizontal table. Horizontal forces of magnitude $2\sqrt{3}N,\ 16N,\ 5N\ \ and\ \ 15N$ act on the body on the bearings of $030^0, 120^0, 000^0\ \ and\ \ 270^0$ respectively. Find the resultant force on the body and the magnitude of the acceleration.

2. A vehicle traveling on a straight horizontal track joining two points A and B accelerates at a constant rate of $0.25\ ms^{-2}$ and decelerates at a constant rate of $1\ ms^{-2}$. It covers a distance of $2.0\,km$ from A to B by accelerating from rest to a speed of $v\ ms^{-1}$ and traveling at that speed until it starts to decelerate to rest.

   (a) Express in terms of v the times for acceleration and deceleration.

   (b) Given that the total time for the journey is 2.5 minutes, find a quadratic equation for

   v and determine v .

3. (a) Given that A and B are two independent events such that $P(A)=0.64$ and

   $P(A\cup B)=0.80$, find : (i) $P(B)$ (ii) $P(A\cup B')$.

   (b) A bag contains 5 red beads, 3 blue beads, and 2 yellow beads. Two beads are drawn from the bag , one after the other, without replacement. Find the probability of drawing:

   (i) two red beads.

   (ii) one blue and one yellow bead.

   (iii) beads of same colours.

   Construct a probability distribution for the number of red beads drawn and find the expected value.

4. During winter a family requests bottles of milk everyday, and they are left on the door steps. The probability distribution of X, the number of bottle tops removed on a given day is given by

$$f(x)=\begin{cases} kx & : x=1,2,3,4,5 \\ k(10-x) & : x=6,7,8,9 \end{cases}.$$

(i) Find the expected number of bottle tops removed on any given day.

(ii) Find $Var(2X-3)$.

5. (a) Two events A and B are such that $P(A)=0.20$, $P(A'\cap B)=0.22$ and $P(A\cap B)=0.18$. Find : (i) $P(A\cap B')$ (ii) $P(A\cup B)'$.

(b) Soldiers A, B and C each fire one shot at a target. The probability that A will hit it is $\frac{1}{5}$, B hits it is $\frac{1}{4}$ and C hits it is $\frac{1}{3}$. If they fire together, calculate the probability that:

(i) all the soldiers' shots hit the target,

(ii) soldier $B's$ shot hits the target,

(iii) exactly two shots hit the target,

(iv) at least one shot hits the target.

6. (a) Two events are such that $P(A)=0.45$, $P(A'\cap B)=0.36$ and $P(A\cap B)=0.25$. Find : (i) $P(A\cap B')$ (ii) $P(B)$ (iii) $P(A\cup B)'$.

7. (a) Use the trapezium rule with five sub- intervals to find the following correct to three decimal places: (i) $\int_1^2 e^{-2x}\,dx$ (ii) $\int_0^1 \frac{1}{1+\sin x}\,dx$.

(b) The table below shows the cost y in shillings for hiring a motorcycle for a distance x kilometers.

x	10	20	30	40
y	2800	3600	4400	5200

Use the linear interpolation or extrapolation to find:

(i) the cost of hiring the motorcycle for a distance of 45 km,

(ii) the distance travelled if one pays sh. 4000.

MERRYLAND HIGH SCHOOL – ENTEBBE
S5 TERM 2020 HOLIDAY BREAK
MATHEMATICS P1
Pure maths

INSTRUCTIONS:
Answer ALL questions

1. Solve the following equations:
   a) 4x – 5(2x) + 4 = 0
   b) 2 – 5e –x + 5e -2x = 0
   c) $x^{\frac{4}{3}} + 16x^{\frac{-4}{3}} = 17$
   d) $x^2 + 2x = 34 + \frac{35}{x^2+2x}$
   e) $\sqrt{2-x} + \sqrt{3+x} = 3$
   f) $3^{2(x+1)} - 10^{(3^x)} + 1 = 0$

2. If $x^2 + x + b = 0$ and $x^2 + ax + b = 0$ have a common root, show that;
   $(b-1)^2 = (a-1)(1-ab)$

3. Solve the following pairs of simultaneous equations
   a) $x^2 + y^2 - 6x + 4y - 13 = 0$
   $x^2 + y^2 - 10x + 10y - 15 = 0$

   b) $4x^2 + 25y^2 = 100$
   xy = 4

   c) Log x – 2 log y = log 2
   x – 5y + 2 = 0

4. a) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$, show that;
   $\frac{a+c+e}{b+d+f} = k$

   b) Hence solve the simultaneous equations.

5. Express the following in surd form and rationalize the denominators.
   a) $\frac{1+\tan 30^o}{1-\tan 30^o}$
   b) $\frac{1}{(1-\sin 45^o)}2$

6. Solve the following equations for;
   $0^o \leq \theta \leq 360^o$
   a) $2\cos^2\theta + 3\cos\theta + 1 = 0$
   b) $3\cos\theta + 2\sec\theta + 7 = 0$
   c) $3 - 3\cos\theta = 2\sin^2\theta$
   d) $2\tan^2\theta + \sec\theta = 1$
   e) $3\sin^2\theta - \sin\theta\cos\theta - 4\cos^2\theta = 0$
   f) 2 sin x = cos (x + 60o)

6. Prove the following identities;
   a) $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$
   b) $\sec\theta + \operatorname{cosec}\theta\cot\theta = \sec\theta\operatorname{cosec}^2\theta$
   c) Sin (A+B) + sin(A–B) = 2 sin A cos B
   d) $2\tan^2\theta + \sec\theta = 1$
   e) $3\sin^2\theta - \sin\theta\cos\theta - 4\cos^2\theta = 0$
   f) $2\sin x = \cos(x + 60^o)$

7. Prove the following identities
   a) $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$
   b) $\sec\theta + \operatorname{cosec}\theta\cot\theta = \sec\theta\operatorname{cosec}^2\theta$
   c) Sin (A + B) = 2 sin A cos B.
   d) $\sqrt{\frac{1-\cos 2x}{1+\cos 2x}} = \tan x$

8. Given that $\log_2 x + 2\log_4 y = 4$. Show that xy = 1.6. Hence solve the simultaneous equations:
   $\log_{10} x + y = 1$
   $\log_2 x + 2\log_4 y = 4$

END

MERRYLAND HIGH SCHOOL – ENTEBBE
S5 TERM 2020 HOLIDAY BREAK
MATHEMATICS P2

INSTRUCTIONS:
Answer ALL questions.

1. The table below shows the marks obtained by 200 students in an examination marked out of 100%

Marks (%)	Number of candidates
10 – 19	18
20 – 29	34
30 – 39	58
40 – 49	42
50 – 59	24
60 – 69	10
70 - 79	6
80 - 89	8

(a) Calculate the;
   (i) mean mark
   (ii) modal mark

(b) Draw a cumulative frequency curve for the data. Hence estimate the lowest mark for a distinction one if the top 5% of the candidates qualify for the distinction.

2. Find the resultant of two vectors of magnitude 5 units, direction 32!Y', and a vector of magnitude 8 units, direction055°.

3. For a set of 20 numbers, $\sum x = 300$ and $\sum x^2 = 5500$. For a second set of 30 numbers, $\sum x = 480$ and $\sum x^2 = 9600$. Find the mean and standard deviation of the combined set of 50 numbers.

4. The mean of numbers; 3, 6, 7, a, 14 is 8. Find the standard deviation of the set of numbers.

5. A class performed an experiment to estimate the diameter of a circular object. A sample of five students had the following results in centimeters; 3.12, 3.16, 2.94, 3.33 and 3.00. determine the sample;

(i) mean
(ii) standard deviation.

6. (a) Find the value of A if $Ai + 2j - k$ and $5i + Aj - k$ are perpendicular vectors.
   (b) Find the angle between a force of 6N and a force of 5N given that their resultant has magnitude 9N.

7. The table below shows the income of 40 factory workers in millions of shillings per annum.

1.0	1.1	1.0	1.2	5.4	1.6	2.0	2.5
2.1	2.2	1.3	1.7	1.8	2.4	3.0	2.2
2.7	3.5	4.0	4.4	3.9	5.0	5.4	5.3
4.4	3.7	3.6	3.9	5.2	5.1	5.7	1.5
1.6	1.9	3.4	4.3	2.6	3.8	5.3	4.0

a) Form frequency distribution table with class intervals of 0.5 million shillings starting with the lowest limit of I million shillings.

b) Calculate the;
i. mean income
ii. standard deviation

c) Draw an Ogive to represent the above data and use it to estimate the median income.

8. The table below shows the amount of money (in thousand of shillings) that was paid out as allowances to participants during a certain workshop.

Amount (sh '000s)	No. of participants
110 – 114	13
115 – 119	20
120 – 129	32
130 – 134	17
135 – 144	16
145 – 159	12

(a) Find the mean, mode and median and standard deviation.
(b) Obtain the interquartile range
(c) Find the 40^{th} and 60^{th} percentile range

9. The table below shows marks obtained by 20 students in a math test marked out of 20

Marks	10	11	12	13	14	15	16	17	18	19
No. of students	1	2	2	2	2	4	2	1	2	1

Using assumed mean of 15, find the mean mode median and standard deviation.

10. Given the following set of values

2, 1, 3, 4, 5, 6, 7, 8, 9, 10, 3, 4, 6, 7, 6, 8, 9, 6, 3, 2

(a) Form a frequency distribution table of un grouped data

(b) Use your table to obtain the: mean, Mode, median,

(c) Find the standard deviation and interquartile range.

11. A bag contains five balls each bearing one of the numbers 1, 2, 3, 4, 5. A ball is drawn from the bag, its number noted and replaced. This was done 50 times in all and the table below shows the resulting frequency distribution.

Number	1	2	3	4	5
Frequency	x	11	y	8	9

If the mean is 2.7, determine the values of x and y.

12. Find the angle between the following pairs of vectors

(i) 5i+j-2k and 4i+3j-8k

(ii) -4i+2j-4k and 2i-j+2k

13. If a $= \begin{pmatrix} 2 \\ 7 \\ 7 \end{pmatrix}$, b $= \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}$, c $= \begin{pmatrix} 0 \\ -4 \\ -3 \end{pmatrix}$

(i) Find the resultant of a and b

(ii) find $|a + b + c|$

(iii) Obtain the value of $|a + 3b - 2c|$.

14. The table below shows delivery charges by DAKS courier.

Mass in (g)	200	400	600
Charges (sh)	700	1200	3000

Using linear interpolation of extrapolation, find the;

a) Mass of a percel whose delivery charge is shs. 3500
b) Delivery charge of a percel weighing 352g

15. Use the trapezium rule with 7 ordinates to estimate $\int_0^3 \frac{1}{1+x}dx$ correct to 3dp.

END